The Use of the Poynting Vector in Interpreting ULF Waves in Magnetospheric Waveguides

T. Elsden, A. N. Wright

Abstract. We numerically model ultra low frequency (ULF) waves in the magnetosphere assuming an ideal, low-β, inhomogeneous plasma waveguide. The waveguide is based on the hydromagnetic box model. We develop a novel boundary condition that drives the magnetospheric boundary by pressure perturbations, in order to simulate solar wind dynamic pressure fluctuations disturbing the magnetopause. The model is applied to observations from Cluster and THEMIS. Our model is able to reproduce similar wave signatures to those in the data, such as a unidirectional azimuthal Poynting vector, by interpreting the observations in terms of fast waveguide modes. Despite the simplicity of the model, we can shed light on the nature of these modes and the location of the energy source relative to the spacecraft. This is achieved by demonstrating that important information, such as phase shifts between components of the electric and magnetic fields and the balance of radial to azimuthal propagation of energy, may be extracted from a careful analysis of the components of the Poynting vector.

1. Introduction

Ultra low frequency (ULF) waves are a dominant feature of Earth’s magnetosphere and have been studied extensively for over half a century. From a theoretical standpoint, Southwood [1974] and Chen and Hasegawa [1974] examined the coupled equations for fast and Alfven magnetohydrodynamic (MHD) waves arising from the cold plasma equations, choosing a uniform background field B0 with a density variation across the field radially in x. The solution reveals the location of a resonance on the field line satisfying ω2(x) = ω2. These studies also showed that surface waves excited by the Kelvin-Helmholtz instability at the magnetopause can couple to a field line resonance (FLR) within the magnetosphere. These ideas were later advanced analytically treating the magnetosphere as a cavity in the hydromagnetic box model of Kivelson and Southwood [1985], and implemented numerically by Allan et al. [1986a], Allan et al. [1986b] and Lee and Lysak [1989]. It was argued that the driving of the cavity from an external source would excite the natural fast mode frequencies of the cavity. These in turn would, at the resonance locations, couple to the Alfven mode resulting in a FLR. More recently, large scale MHD simulations have been performed to attempt a more realistic modeling of magnetospheric cavity modes [e.g. Claude-pierre et al., 2009], where it is shown that these modes can be driven by solar wind dynamic pressure fluctuations.

A drawback to the cavity model is that only discrete frequencies can be obtained due to the quantization of the wavenumber ks, since the cavity is treated as axisymmetric. This model was refined to the waveguide model of the magnetosphere [e.g., Harrold and Samson, 1992; Wright, 1994], treating it as open ended to account for the structure of the magnetotail. To this end, much analytical and numerical work has been done in the papers Wright [1994], Wright and Rickard [1995a], Rickard and Wright [1994,1995] as an effort to further explain the coupling between the fast and Alfven modes.

Observationally, the field of magnetospheric ULF waves has progressed drastically with the availability of a plethora of satellite data, compared to the times when most of the underpinning theory was developed. This permits a much more detailed comparison between the theory and the observations. In this study, we retain the numerical simplicity of the waveguide model of Rickard and Wright [1994] whilst attempting to match to recent observations. Two observational cases are considered, firstly that of Clausen et al. [2008] reporting evidence of ULF waves from Cluster data, and secondly that of Hartinger et al. [2012], discussing a Pc5 global mode observed by the Time History of Events and Macroscale Interactions (THEMIS) satellite. Henceforth, for ease of reference, these papers shall be referred to as Clausen08 and Hartinger12 respectively.

In the model, we employ a new driven boundary condition where the magnetopause boundary is driven with perturbations of the field aligned magnetic field component b, as a means to mimick driving with pressure, which Takanashi and Ukhorskiy [2008] suggest is the dominant driver of magnetospheric ULF waves. This differs to previous simulations where displacement was used to drive the system [e.g. Wright and Rickard, 1995b].

The Poynting vector components are of particular interest for interpreting the energy flow within the domain, and we have been used in the analysis of ULF waves. Junginger [1985] provides both an analytical and numerical treatment of the time averaged Poynting vector in a box geometry in the presence of mode coupling. This theory is supported by Proehl et al. [2002], where a 3D MHD simulation shows time averaged radial energy diverted into the resonant surface. Cummings et al. [1978] used the Poynting vector to identify azimuthal propagation of a Pc4 wave. Chi and Russell [1998] found a correlation between the time-averaged Poynting flux and phase skips, a phenomenon where the phase changes suddenly between wave packets with a near constant frequency. As in these studies, we demonstrate how the Poynting vector can be used to interpret ULF wave signatures.

The paper is laid out in the following way: Section 2 discusses in detail the aforementioned observations as a means to motivate our modeling. Section 3 develops the theory surrounding the waveguide approach, listing the governing equations, boundary conditions and numerical method. Sections 4 and 5 present the results from the modeling of...
the events from Clausen08 and Hartinger12 respectively. A comparison between the papers and some final thoughts are given in section 6. Appendix A discusses in depth how the new driven boundary condition on the magnetopause affects the nodal structure and frequency.

2. Observations

2.1. Cluster Observations

We begin by considering Clausen08 which discusses a large scale Pc4 pulsation observed by ground magnetometers and the Cluster satellites. Figure 1 of Clausen08 shows a clear wavepacket signature in the ground magnetometer data most prominently in the dawn sector, but also visible at magnetic local time (MLT) $\sim 14$, from 13:30 - 13:42. Stations spanning longitudes from $\sim 50^\circ$ to $\sim 75^\circ$ also pick up the signal, showing that the event covered a large radial distance in the magnetosphere. Cluster 3 and 4 sample roughly the same set of fieldlines at $\sim 8$ MLT. During the event, both Cluster 3 and Cluster 4 remained relatively close to the magnetic equator, Cluster 3 between $9.6^\circ$ and $14.7^\circ$ magnetic latitude with Cluster 4 between $-12.0^\circ$ and $-7.8^\circ$. Clausen08 determined the location of the plasmapause to be at a radius of $4.1 R_e$. This indicates that both satellites sit very close to the plasmapause which will be key to their position in our modeling. The satellite data for Cluster 3 and 4 are qualitatively similar, and we reproduce the Cluster 3 data in Figure 1 (taken from Figure 3 of Clausen08). The left hand panels list the 3 components of the magnetic field $b$ (top), electric field $E$ (middle) and the Poynting vector $S$ (bottom) calculated from

$$S = \frac{1}{\mu_0} E \times b,$$

where the above fields are the wave or perturbation fields. We shall concentrate on the main wavepacket occurring from 13:30 - 13:42. The right hand panels display the corresponding FFT, showing the dominant signal at 17.2 mHz for the magnetic and electric field components, with twice the frequency as expected for the Poynting vector signals. It is remarkable that the Cluster electric and magnetic field data has not been filtered, but is a genuinely coherent oscillation with a well defined natural frequency. Considering the components, comparable field aligned and azimuthal magnetic field perturbations of around 1 nT are observed, with negligible radial signal. We note that the growth period of the perturbation is approximately 4 periods (estimated from the $b_z$ data; this will become important for modeling in terms of how long to drive the system). For the electric field we see a strong radial component, a small azimuthal component and negligible field aligned variation. The Poynting vector calculated from these components has an oscillatory field aligned signature with no preferred direction showing equal energy transfer between the northern and southern hemispheres. Perhaps the most interesting feature, is the azimuthal Poynting vector signature showing a purely unidirectional flow of energy into the tail. This feature will be examined in detail in our simulations. As expected from a negligible radial magnetic field and small azimuthal electric field, there is only a small radial Poynting vector signature.

2.2. THEMIS Observations

The second observation that we consider was reported by Hartinger12. In this paper the authors discuss the observation of a global mode, described as ‘standing fast mode waves trapped between different magnetospheric boundaries’. Spacecraft data from THEMIS is presented along with a figure showing satellite data from THD.

![Figure 1](image1.png) Reproduction of Figure 3 from Clausen08 displaying data from the Cluster 3 satellite. On the left: the 3 components of the magnetic field (top panels), electric field (middle) and the Poynting vector (bottom). On the right are given the corresponding fast Fourier transform powers.

![Figure 2](image2.png) Reproduction of Figure 7 from Hartinger et al. [2012], showing satellite data from THD.
with data from ground magnetometer stations. THEMIS consists of five satellites THA-THE, that in this case were able to observe solar wind, magnetosheath, magnetospheric and plasmaspheric plasma. The dominant driver in this case is broadband fluctuations of solar wind dynamic pressure. The location of the plasmapause is estimated at ∼ 7 R$_e$, whilst the dawn magnetopause is at a radial distance of ∼ 17 R$_e$.

The main components that we explain are given in Figure 7 of Hartinger12, and reproduced here as our Figure 2. The components are given in field aligned coordinates for the THD spacecraft, with $z$ being the field aligned component, $y$ positive eastwards and $x$ outwardly. THD is situated at an L-value of ∼ 8.5 and MLT ∼ 6 hours, and is hence just on the magnetospheric side of the plasmapause on the dawn flank. Furthermore, THD lies within 1 R$_e$ of the magnetic equator. The components shown in Figure 2 are band pass filtered between 5 mHz and 8 mHz in order to isolate the global mode frequency of 6.5 mHz. Strong components $b_y$ and $E_y$ are observed shown in panels 1 and 2 (along with the high pass filtered data, > 0.5 mHz), with weak $b_x$, $b_z$ and $E_z$ shown together in panel 5. It is expected to see weak $b_x$ and $b_y$ components for the satellite location, close to the magnetic equator since assuming a fundamental field line structure in the $z$ direction, these components have a node here. The Poynting vector signal composed from the magnetic and electric field components has a strong radial component which is predominantly inward, recovering to a $x$ component which is predominantly outward, according to the near equatorial position. These Poynting vector signatures are very different to those from Cluster, shown in Figure 1, and will be used to learn about the nature and location of the source of wave energy.

2.3. Observation Selection and Modeling Goals

The event outlined in Clausen08 was selected for modeling mainly due to the Poynting vector signature showing purely tailward propagation of energy. The second event described by Hartinger12 was chosen as a contrast to the Cluster observation in order to compare the two different signatures. The reported global mode shows very little azimuthal Poynting vector signal, with a negligible field aligned component as expected by the near equatorial position. These Poynting vector signatures are very different to those from Cluster, shown in Figure 1, and will be used to learn about the nature and location of the source of wave energy.

3. Model

We model the flank of the magnetosphere using a waveguide in a Cartesian system, where $x$ points radially outward, $y$ is the azimuthal coordinate around the earth and $z$ is field aligned, assuming a uniform background field $B = B_0 \hat{z}$. This is in a similar fashion to the hydromagnetic box model of Kivelson and Southwood, [1986]. A schematic of the model is given in Figure 3. In order to model a waveguide rather than a cavity, we assume an infinite length in the $y$ direction. In practice this condition is realised by ensuring that the grid is long enough such that over the course of the simulation, no disturbances reach the far boundary in $y$. The density $\rho$, is taken to be a function of $x$ only, such that the Alfven speed varies with radius. We assume a cold (low $\beta$), ideal plasma that has perfectly reflecting boundaries in $x$ and $z$, except over the driven portion of the magnetopause.

The inner boundary in $x$ is assumed to be the plasmapause with the outer boundary being the magnetopause. In $z$, a fundamental standing mode structure is assumed with nodes of $u$ (velocity) at the ionospheric ends of the fieldlines. Hence we only need to consider a fundamental $k_z$ Fourier mode dependence. Choosing the magnetic equator to be situated at $z = 0$, we let $u_x, u_y, b_z \sim \cos(k_z z)$ and $b_x, b_y \sim \sin(k_z z)$. Working from the equations of induction and motion our system is then given by the dimensionless equations

\[
\frac{\partial b_x}{\partial t} = -k_z u_x,
\]

\[
\frac{\partial b_y}{\partial t} = -k_z u_y,
\]

\[
\frac{\partial b_z}{\partial t} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y},
\]

\[
\frac{\partial u_x}{\partial t} = \frac{1}{\rho} \left( k_z b_y - \frac{\partial b_z}{\partial x} \right),
\]

\[
\frac{\partial u_y}{\partial t} = \frac{1}{\rho} \left( k_z b_x - \frac{\partial b_z}{\partial y} \right).
\]

where we have normalised the magnetic field by the equilibrium magnetic field strength $B_0$, the density by $\rho(0)$ (at the plasmapause) and length by the width of the waveguide in $x$ (See Wright and Rickard, [1995b] equations (1a)-(1e)).

The variation of the Alfven speed with radius is given by the piecewise function

\[
V_A(x) = 1 - \frac{x}{x_0}, \quad 0 < x \leq x_e
\]

\[
V_A(x) = \frac{x_0 \left( 1 - \frac{x}{x_0} \right)}{(x_0 - 2x_e + 1)(1 - x_e) - (1 - x)^2}, \quad x_e < x < 1
\]

as in Wright and Rickard, [1995b]. The point $x_e$ defines the position where the profile switches and $x_0$ defines the gradient of the profile. The above profile has the advantage that in the region $0 < x \leq x_e$, the Alfven speed is linear in $x$, and hence the phase mixing length is independent of $x$. This allows for efficient computation by minimizing oversolution in the domain.

To integrate the system of equations given by (1)-(5) we use a leapfrog-trapezoidal algorithm, the details of which are in Wright and Rickard, [1994] (equations (13-15)). The grid is of length 1 (10 R$_e$) in the radial ($x$) direction from $x = 0 \rightarrow 1$ and 10 (100 R$_e$) in the azimuthal ($y$) direction from $y = 0 \rightarrow 10$. As mentioned previously, this length

\[
\text{Figure 3. Magnetospheric waveguide geometry. Moving in } y \text{ corresponds to azimuthal propagation around the dusk flank and the driving region indicated can either model noon or flank driving.}
\]
in \( y \) is chosen such that a disturbance will not reach the far boundary. The assumption of perfectly reflecting boundary conditions in \( x \) is accomplished by setting \( u_x \) and \( u_y \) to zero at the inner radial boundary. We impose a symmetry condition on the \( y = 0 \) boundary which models the center of the driven section of the magnetopause. This could correspond to the subsolar point for symmetric dayside driving, or the flank depending upon the driving mechanism and prevailing conditions. This is realised by setting \( u_x \) and \( b_y \) to zero on this boundary. 200 grid points are taken in both the \( x \) and \( y \) dimensions with a timestep of \( \Delta t = 0.001 \) (0.027 s). We ensure that the CFL condition is satisfied and the phase mixing length resolved over the course of the simulation [Mann et al., 1995; Rickard and Wright, 1994]. For the simulations reported in this paper, energy conservation was tested and was met to 1 part in \( 10^3 \) or better.

In order to drive the system, we prescribe a time dependent profile to the aligned magnetic field perturbation \( b_z \) over a portion of the outer \( x \) boundary (magnetopause). We tailor this to the observation in question (see sections 4.1 and 5.1 for details). This mimicks driving with pressure which is posited as the dominant driver of magnetospheric ULF waves [Takahashi and Ukhorskiy 2008]. Driving in this way differs from the previous model in this area by Clausen08, Clausen08, Wright and Rickard [1994], Wright and Rickard [1995b], where the system is driven by a prescribed displacement in the \( x \) direction, \( \xi_x \). Driving with displacement simulates a node of \( u_z \) at the outer boundary, which together with a node of \( u_z \) at the inner boundary would imply that the fundamental is a half wavelength radial mode. However changing to drive with \( b_z \) simulates a node of \( b_z \) at the outer boundary (see Appendix A), and hence an antinode of \( u_z \). This implies that the radial fundamental for the new driving regime is a quarter wavelength mode. Adopting such a boundary condition is in agreement with Mann et al. [1999], who suggested that over-reflected waveguide modes believed to drive discrete FLRs are more accurately modeled by such a boundary condition. They also noted that having a fundamental quarter wavelength mode can help to lower the natural waveguide eigenfrequencies to just a few mHz without changing the magnetospheric plasma density to unrealistic higher values. Claudepierre et al., [2009] also came across this problem in their simulation looking at magnetospheric cavity modes driven by solar wind dynamic pressure fluctuation. They found that adopting a quarter wavelength mode would much better suit the frequencies for the cavity mode.

In the following two sections we choose simulation parameters appropriate to the Cluster and THEMIS events described in section 2. We then experiment with driving conditions and satellite locations in an effort to reproduce notable signatures in the observations.

4. Cluster Modeling

4.1. Tailoring the Model

In order to model the Cluster observations we use specific input parameters given in Clausen08. Figure 10 from their paper displays a model of how the fundamental field line frequency will vary with L-shell. As mentioned above, we take the inner boundary of the waveguide to be the plasmapause at \( \sim 4 R_E \), and allow a radial extent (in \( x \)) of 10 \( R_E \) to the magnetopause at \( \sim 14 R_E \). Using their Figure 10, we determine that the Alfvénic frequency varies from \( \sim 12 \) mHz at the plasmapause to \( \sim 5 \) mHz at the magnetopause. Assuming that the Alfvén speed is proportional to the Alfvén frequency, this allows the Alfvén speed profile to be scaled to match this frequency change. We note that these frequencies are a little high for typical fundamental Alfvén modes, probably due to lower plasma densities than normal which have elevated the normal Pc5 frequencies to the Pc4 band. Indeed, Clausen08 suggest that it is due to the natural frequencies in this event matching the frequency generated by back-streaming ions at the bow shock that the lowest frequency modes of the magnetosphere can be excited effectively by this method [Le and Russell, 1996].

The system is driven with the \( b_z \) perturbation as described in section 3, with a frequency of 17.2 mHz to match the dominant frequency in the Cluster data in Figure 1. Le and Russell, [1996] developed a model for the frequency of the mode generated by back-streaming ions at the bow shock, formulating the frequency in terms of the cone angle and the interplanetary magnetic field strength. Figure 8 from Clausen08 shows that these parameters were relatively stable over the course of the event from 13:30 to 13:40 UT. Furthermore, even broadband frequency driving can give monochromatic signatures in a cavity or waveguide [Wright and Rickard, 1995a]. Both of these effects justify driving monochromatically. Our equilibrium model is chosen such that the driving frequency of 17.2 mHz is the second radial harmonic of the waveguide. Considering the radial fundamental to be a quarter wavelength mode, with the prescribed boundary conditions the second harmonic has approximately 3 times the frequency of the fundamental. This implies a fundamental frequency of \( \sim 6 \) mHz.

In order to choose the wavenumber in the field aligned direction \( k_z \), we again refer to the Alfvén frequencies given in Figure 10 of Clausen08. Through the Alfvénic dispersion relation \( \omega_A = k_z V_A \), \( k_z \) can be adjusted with \( V_A \) in order to match both the expected fundamental frequency profile and the full dispersion relation of fast waveguide modes. This is done in practice by solving equations (1)-(5) neglecting variations in the \( y \) direction i.e. with \( k_y = 0 \), using a fourth order Runge-Kutta method to shoot for the waveguide eigenfrequencies. From this come the finalised values for the \( k_z = 0 \) fast natural waveguide frequencies, the field aligned wavenumber \( k_x \) and the correctly scaled Alfvén speed profile. (From this analysis we find dimensionless values of \( k_x = 2.28, x_0 = 1.514 \) and \( x_0 = 0.8 \). To get these in terms of \( R_E \), multiply \( x_0 \) by 10, and divide \( k_x \) by 10. The normalizing magnetic field and velocity were taken as 90 nT and 2350 kms\(^{-1} \), respectively, with an inner boundary Alfvén speed of \( 2107 \) kms\(^{-1} \).) The theory of estimating the natural waveguide frequencies by looking at the \( k_y = 0 \) modes was developed by Wright [1994] and tested in simulations by Rickard and Wright [1994].

The length in the \( z \) direction is now fixed by the choice of \( k_z \). For the assumption that the mode has a fundamental structure in \( z \). This is consistent with the observed phase shift of \( 180^\circ \) in the azimuthal magnetic field component \( b_y \) between Cluster 3 and 4. This infers that the satellites must straddle a node of \( b_y \), one above and one below the magnetic equator. Results from a preliminary simulation using the above input parameters show that assuming a homogeneous medium in the \( z \) direction, at a satellite location modeling that of Cluster 3 (\( x = 0.05, y = 0.6 \) and \( z = 0.1 \) in dimensionless units), the \( b_y \) component has too small an amplitude in comparison to \( b_z \). The \( b_y \) component is small in the simulation due to the position of both satellites near to a node of \( b_y \) at the equator. Including an inhomogeneity in the density profile, \( n(z) \), would create a dependent Alfvén speed which could shift the turning point of the mode towards the equator. The low Alfvén speed in this equatorial region tends to cause the mode's phase structure to bunch up there, as seen in Figure 12 of Clausen08 showing the Alfvén eigenfunctions. This is important as it would allow for higher harmonics closer to the equator, and could hence give a better match to the observed amplitudes.

To see the effects of including such an inhomogeneity, we solve equations (1)-(5) for a fast mode in a Cartesian geometry using a Runge-Kutta 4th order method, over the full field line length, neglecting variations of the equilibrium in the
parallel field direction and imposing a $z$ dependence on $\rho$ such that $\rho = \rho(z)$. Furthermore, we assume a dimensionless perpendicular wavenumber of $k_{\perp} = 4.71$ (which could correspond, for example, to a radial wavelength of $\lambda_{\parallel} = \frac{3}{4}$ with $k_{\parallel} = 0$). The increase of $V_3$ along a field line for a dipole field is mimicked by allowing $\rho(z)$ to vary accordingly. Figure 4 gives the nodal structure of the dependent variables plotted against distance along the field line, in a similar manner to Figure 12 from Clausen08. The approximate locations of the Cluster satellites are denoted by the vertical dashed lines, and it is first of all evident that we see a 180° plotted against distance along the field line, in a similar manner 4 gives the nodal structure of the dependent variables $\lambda$ respond, for example, to a radial wavelength of $\perp$ and imposing a perpendicular wavenumber of $\perp$ such that $\perp = 4$ demonstrates that the Cluster satellites would lie close to the antinodes of $b_3$. Hence moving closer to the antinodes in our original homogeneous $z$ structure, should simulate the amplitudes that would be found closer to the equator when there is inhomogeneity in $z$.

### 4.2. Results

Figure 5 shows the time dependence of $b_2$ on the outer driven boundary at $x = 10 R_e$. The system is driven for 4 periods of 17.2 mHz to match the observed growth phase of the $b_2$ perturbation in the Cluster 3 data and after four cycles the driver is switched off. The spatial dependence of $b_2$ along the outer boundary at $x = 10 R_e$ in the $y$ direction is given in Figure 6. The extent of the disturbance on the magnetopause boundary i.e. the size of the driven region corresponds to a physical length of $5 R_e$ (and a full width of $10 R_e$). The satellite position for Cluster 3 is taken to be at 4.5 $R_e$ in $x$ and 6 $R_e$ in $y$. These are taken as approximations to the real location of the satellite and considering the geometric simplifications of our model. This corresponds to dimensionless coordinates $x = 0.05$ and $y = 0.6$, recalling that the inner boundary of the waveguide at $x = 0$ corresponds to a radial position of 4 $R_e$. The length of a fieldline in $z$ is determined by the choice of $k_{\parallel}$. For the given input parameters the length in $z$ is 1.378 and hence extends from $-0.689 \rightarrow 0.689$, corresponding to a dimensional length of almost 14 $R_e$. The perpendicular components of the magnetic field have antinodes at the ends of the fieldlines. As discussed above, moving towards the antinodes of $b_3$ should simulate the amplitudes that would be observed if an inhomogeneity in $z$ was considered. Hence the position in $z$ is chosen as 0.45 approximately two thirds along the length of the fieldline.

Figure 4. $u_\perp$, $b_\perp$ and $b_2$ (solid lines) plotted against distance along the field line for a ‘dipole’ inhomogeneity in $z$, with the dashed line $b_\perp$ for a homogeneous medium in $z$. The vertical lines show the approximate location of the Cluster satellites, approximately $2 R_e$ apart.

Figure 5. Temporal variation of the $b_2$ driver on the driven boundary $x = 10 R_e$ at $y = 0$.

Figure 6. Spatial variation in $y$ of $b_2$ on the driven boundary $x = 10 R_e$. 

The heightened amplitudes during driving extend past the driver switch off time by approximately the radial travel time ($t \sim 1.04$ minutes, shown by the second vertical dashed line), since the satellite is located close to the inner boundary. As in the observations, the field aligned and azimuthal magnetic field components have similar amplitudes, shown in panels (a) and (b) respectively. Notice also how $b_2$ falls
off as soon as the driving stops (plus radial travel time) at around \( t = 5 \) minutes. The field aligned component is persistent post driving, which is a result of driving on resonance: as we drive with the natural frequency of the second radial waveguide harmonic, no other modes are excited to the same extent, and hence we see a clear monochromatic response after the driving has stopped. The radial component \( b_x \) given in panel (c) is small due to the position of the satellite close to the inner boundary (plasmapause), where \( b_x \) has a node. Through the equations and the simplifications made, only the perpendicular components to the velocity remain and are shown in panels (d) and (e). Recall that the electric field has been eliminated in favour of the velocity field in our model. For the purpose of comparison the components may be identified as \( u_x \sim E_y \) and \( u_y \sim -E_x \). The position of the satellite close to the perfectly reflecting inner boundary where \( u_x = 0 \) causes the radial component of velocity (and \( E_y \)) to be small. The azimuthal component of velocity \( u_y \) matching the simulation with strong \( u_y \) and weak \( u_x \).

A diagnostic of energy flow within the system is the Poynting vector, the components of which are given in panels (f)-(h). The field aligned and azimuthal signatures dominate, with the radial component being weaker, as in the case of \( b_x \) and \( u_y \), due to the near inner boundary position. The field aligned component of \( S \) shows equal parallel to antiparallel transport of energy, as is expected for modes that stand in \( z \) in the absence of ionospheric dissipation. The azimuthal component matches well to the striking result from Figure 1, where we clearly see the purely tailward (positive azimuthally in our model) propagation of energy. Since the perpendicular magnetic and velocity field components have a decreased amplitude signal post driving, this feature is seen in the Poynting vector signal as well.

\textit{Clausen08} report certain phase shifts in the electric and magnetic field components at each Cluster satellite and between the satellites. Firstly, the observations display a 180° phase shift in \( b_y \) between Cluster 3 and Cluster 4. This phase shift indicates that the modes have a fundamental standing structure in \( z \), with Cluster 3 above the magnetic equator and Cluster 4 below. This property is clearly seen in the top panel of Figure 8, where the simulation position of Cluster 4 is taken to be \( x = 0.05, y = 0.6 \) and \( z = -0.45 \). Also present in the observations is a 90° phase shift between the field aligned and azimuthal magnetic field components.

Figure 7. Components of the magnetic field (panels (a)-(c)), velocity field (panels (d) and (e)) and the Poynting vector (panels (f)-(h)) at \( x = 0.05, y = 0.6 \) and \( z = 0.45 \), modeling the position of Cluster 3 from \textit{Clausen08}. The first vertical dashed line at \( t = 3.88 \) minutes is the time when the driver is switched off and the second at \( t = 4.92 \) minutes, includes the radial travel time.

Figure 8. Simulation results matching observed phase shifts. Top: \( b_y \) from Cluster 3 (black) with \( b_y \) from Cluster 4 (red); middle: \( b_y \) from Cluster 3 (black) with \( -u_y (E_x) \) from Cluster 3 (red); bottom: \( b_z \) from Cluster 3 (black) with \( -u_y (E_x) \) from Cluster 3 (red).
at any given satellite. The second panel of Figure 8 shows the phase shift between these components at Cluster 3 and indeed matches well to the observations. Finally, Clausen08 observe a 180° phase shift between $E_x$ and $b_z$, which is reproduced from the simulation in panel 3 of Figure 8. As we have chosen to eliminate the electric field for the velocity field, the plot gives the negative azimuthal velocity which is associated with the radial electric field.

4.3. Discussion

Clausen08 interpret their data as a waveguide mode coupling to two FLRs. These were identified in ground magnetometer data and correspond to a fundamental mode at $L \sim 2 - 3$, and a second harmonic at $L \sim 8 - 10$. Note that Cluster (at $L = 5$) is not expected to observe either of the FLRs. Indeed, we suggest that the Cluster observations provide a rather clean observation of a resonantly excited waveguide mode. There are several respects in which the satellite data does not fit well with an Alfvén wave interpretation. The strong $b_z$ component in the observations, of similar amplitude to the $b_y$ perturbation, is not usually associated with an Alfvén wave. If a fast mode was driving an Alfvén resonance one would expect to see persistent signals in $b_y$ and $E_z$ post driving until damped through ionospheric dissipation. Here however, these components are very closely correlated with $b_z$. Furthermore, the expected resonance position can be estimated in the simulation assuming a driving frequency similar to the $k_y \sim 0$ modes is responsible for the driving. As in Figure 4 from Wright and Rickard [1995b], we can plot the resonance position as a function of the density parameter $x_0$. For the given density structure the resonance position does not exist within the domain, suggesting that at no point does the Alfvén frequency at Cluster match the fast mode driving frequency. This conclusion can also be drawn from Figure 10 of Clausen08, where the fundamental field line frequency is plotted against radial distance. The driving frequency of 17.2 mHz does not lie within the magnetospheric portion of the plot (it is inside the plasmasphere), suggesting that this frequency will not match any field line fundamental frequency within the waveguide domain. (See the earlier comments regarding FLR locations deduced from magnetometer data.)

The simulation provides very similar results to the Cluster observations in terms of a fast waveguide mode. The main factor responsible for the form of the signal is the satellite position relative to the driving region. The purely tailward azimuthal Poynting vector signal can be explained by being tailward of the driving region. Fast mode energy enters the waveguide and the larger $k_y$ modes will propagate downtail, whilst the small $k_y$ modes will remain close to the $y = 0$ boundary. Other simulations (not shown here) demonstrate that if the satellite sits within the azimuthal extent of the driver, signals travelling sunward (negative $S_y$) and tailward (positive $S_y$) can be detected, since a fast mode source element creates a disturbance which propagates in all directions. Being further downtail than the driven region however, only signals travelling downstream can be recorded. We believe this to be the simple explanation of the azimuthal Poynting vector signature: a downstream observation of a fast mode source.

The observed phase shifts can be explained in terms of a simple analytic solution considering a Cartesian geometry with propagation in $y$ and a standing mode in $x$ and $z$ for a uniform density as an illustration. This yields the components

\[ u_y \sim \cos(\omega t - k_y y) \cos(k_x x) \sin(k_z z), \]
\[ b_y \sim \sin(\omega t - k_y y) \cos(k_x x) \sin(k_z z), \]
\[ b_z \sim \cos(\omega t - k_y y) \cos(k_x x) \cos(k_z z). \]

It is clear that the phase shifts depicted in Figure 8 match those from the above components. The 180° phase shift between $E_x$ and $b_z$ at Cluster 3 or as in our model $u_y$ and $b_z$ being in phase, determines the unidirectionality of the azimuthal Poynting vector. Hence a Poynting vector signature of this type will always coincide with this phase shift. The 90° phase shift between $b_y$ and $b_z$ again at Cluster 3 is also another signature of propagation in the $y$ direction.

5. THEMIS Modeling

5.1. Tailoring the Model

We now adjust our model to match the situation appropriate to the THEMIS data reported in Figure 2. We take a waveguide of width 10 $R_e$ spanning from $L \sim 7$ to $L \sim 17$, modeling from the plasmapause to the magnetopause. Using the values quoted for the Alfvén speed at $L \sim 8$ from Hartinger12, we take the Alfvén speed at the plasmapause, to be $V_A(0) = 1200$ km s$^{-1}$. This value together with the normalising length scale of 10 $R_e$ (the width of the waveguide) defines a timescale for the model of $T_0 = 53.1$ s. With these values, the dimensional frequency of 6.5 mHz for the
global mode can be converted into a dimensionless angular frequency of $\omega = 2.1683$. As discussed in section 3 we drive the system with the $b_z$ perturbation, assuming a quarter wavelength mode as the 1st harmonic in the radial direction. Hartinger12 had difficulty resolving the radial structure in wavelength mode as the 1st harmonic in the radial direction. Hence we are assuming that the magnetopause is driven on the flanks in this event, rather than at the subsolar point as this event corresponds to a MLT of 6 hours. The $x$ position is taken to be $x = 2 R_e$, modeling the location of THD approximately 2 $R_e$ outside of the plasmapause. The location in $z$ near to the magnetic equator is chosen as $z = 1 R_e$.

5.2. Results

To simulate the observed signal, rather than driving with a broadband signal and filtering the data for the 6.5 mHz signal, the system is driven directly with the global mode frequency. We choose to drive with 5 cycles of $b_z$, in order to match the number of observed driving periods in Figure 2. The magnetopause boundary is driven over an extent of 5 $R_e$ (0.5 in dimensionless units). The temporal and spatial variation of the driver are shown in Figures 9 and 10.

The components of the magnetic field, velocity field and Poynting vector from the simulation at a satellite with position $x = 2 R_e$, $y = 1 R_e$ and $z = 1 R_e$ modeling the location of THD are displayed in Figure 11. The first vertical dashed line demarcates the time when the driver is switched off, at $t = 12.82$ minutes, with the second adding on the radial travel time of $\sim 2.03$ minutes. This second line matches well to the amplitude decrease of the components. The small amplitudes of $b_y$ and $b_z$ in panels (b) and (c) can be attributed to the close proximity to the magnetic equator, where these components have nodes. The field aligned magnetic signature in panel (a) dominates with increasing amplitude over the driving period, changing to a decaying amplitude signal post driving. The persistence of a coherent monochromatic signal once the driver has been switched off is due to the system being driven at the second radial harmonic frequency. This precludes the appearance of other frequency modes in the data and therefore post driving, the waveguide reverberates with this natural frequency. Overall, the magnetic field component results from the numerical model favourably to the components shown in panels 3 and 5 from Figure 2. For the velocity field, panel (e) shows a strong radial component corresponding to the negative of the azimuthal electric field ($E_y$ in panel 2 of Figure 2). The signal is similar in structure to the field aligned magnetic field, with increasing amplitude during driving leading to a gently decaying oscillation post driving. The azimuthal component in panel (d) is small in good agreement with the radial electric field ($E_y$) from the real data. Finally for the Poynting vector components, the nodes of $b_y$ and $b_z$ at the magnetic equator translate to a node of $S_z$, resulting in a negligible

Figure 11. Components of the magnetic field (panels (a)-(c)), velocity field (panels (d) and (e)) and the Poynting vector (panels (f)-(h)) at $x = 2 R_e$, $y = 1 R_e$ and $z = 1 R_e$, modeling the position of THD from Hartinger12. The vertical dashed lines mark firstly the time where the driver is turned off, at $t = 12.82$ minutes and secondly the radial travel time of $t = 2.03$ minutes added to the driver switch off time.

Figure 12. Phase comparison of $b_y$ and $u_y$ at $x = 2 R_e$, $y = 1 R_e$ and $z = 1 R_e$. The dotted line indicates the time when the driver is switched off with the addition of the radial travel time, giving $t = 14.85$ minutes.
field aligned component shown in panel (f). Panel (g) gives the azimuthal Poynting vector, which is small in amplitude and marginally tailward (positive) during the driving period. Post driving, the signal decays rapidly. The radial Poynting vector given in panel (h) is almost entirely inward until the driver is switched off, at which point the signal recovers to a back and forth flow of energy.

A phase shift of 90° between $b_z$ and $E_y$ is expected for a radially standing global mode [Waters et al., 2002]. Figure 12 shows the time signals of $b_z$ and $u_x$ ($E_y$) at the position of THD, with the vertical dashed line marking the radial travel time added to the driver switch off time. Up until this point (during driving), $b_z$ and $u_x$ have a phase difference between 90° and 180°. Post driving, a very clear change in the phase occurs, with the signals being almost exactly 90° out of phase.

5.3. Discussion

As with the previous study of Clausen08 we show that the observational results of Hartinger12 can be reproduced accurately by our numerical simulation. The key to a good match is identifying an appropriate satellite location. The small azimuthal Poynting vector signal, in contrast to the Clausen08 study, tells us that THD must have an azimuthal location that is close to the middle of the driven section of the magnetopause. This is a means of inferring the source location in reference to the satellite position i.e. the centre of the energy source lies approximately on the same flank meridian as THD.

The observed signal from Hartinger12 is believed to be a global mode, standing in the radial direction. The overall inward $S_x$ shown in panel 6 Figure 2 suggests that energy is lost either downtail or through the inner boundary at the plasmapause during the driving phase. The azimuthal Poynting vector signal is small in comparison to the radial component and would at first sight suggest that the energy does not leak out down the tail. The same structure is seen in the simulation however, with a slightly more inward radial Poynting vector, yet the simulation allows for no energy to leak out of the inner boundary (which is treated as perfectly reflecting) or to be coupled to a FLR earthward of the spacecraft. This can be confirmed by considering the energy continuity equation

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} = 0,$$

where $W$ represents the energy density. Consider a small area in the computational domain, from 0 to a in $y$ and from 0 to $b$ in $x$. Integrating in space over this area yields

$$\frac{d}{dt} \int_0^a \int_0^b W \, dx \, dy + \int_0^a S_y \, dy + \int_0^b S_x \, dx = 0.$$

Performing the above calculation for the duration of the run for the THEMIS simulation with $a = 0.1$ and $b = 0.2$ (given in dimensionless units matching the $y$ and $x$ locations of THD), we find that the first term accounts for ~1% of the sum. This results in the inward flow of energy being balanced by the flow of energy downtail, despite a cursory inspection of the data suggesting the net inward flow of energy may be balanced by the increasing wave amplitude during driving. This confirms that a net inward $S_x$ can result without the need for a leaky inner boundary or energy loss to a FLR.

The phase comparison between $b_z$ and $u_x$, shown in Figure 12 highlights the difference between the driving and post driving phases. During the driving phase, there is an overall inward propagation of energy as evident from the shape of the radial Poynting vector $S_x$ [Chi and Russell, 1998], and hence the phases do not adhere to the radial standing mode phase regime of $b_z$ and $u_x$ being 90° out of phase, but instead are phase shifted by between 90° and 180°. Post driving the phase shift changes to 90° and $S_y$ returns to an equally inward and outward signal. To try to better understand the relation between the inward radial Poynting vector signal $S_x$ and the observed phase shifts, we consider the simplest means of describing the signals in Figure 12: two sinusoidal curves with a phase shift, expressed as

$$u_x = \sin(\omega t),$$

$$\frac{b_z}{b_0} = \sin(\omega t + \phi),$$

where $\phi$ is the phase by which $b_z$ leads $u_x$. Constructing $S_x$ yields

$$S_x = u_x b_z,$$

$$\frac{S_x}{u_0 b_0} = \frac{1}{2} \cos \phi + \frac{1}{2} (\sin \phi \sin(2\omega t) - \cos \phi \cos(2\omega t)).$$

The second term on the right hand side can be expressed as a single sinusoid which gives

$$\frac{S_x}{u_0 b_0} = \frac{1}{2} \cos \phi + \frac{1}{2} \cos(2\omega t + \phi_1),$$

where $\phi_1$ is the new phase dependent on $\phi$. Chi and Russell [1998] give plots of $S_x$ for the two limits $\phi = 0$ (propagating) and $\phi = \pi/2$ (standing). The above equation is valid for intermediate cases too. For a net inward energy flow, 90° < $\phi < 270°$. The quantity important for linking the shape of the radial Poynting vector $S_x$ to the phase shift is the ratio of positive to negative $S_x$ signal. This is defined as the absolute value of the maximum outward $S_x$ to the maximum inward $S_x$:

$$\Delta_x = \left| \frac{\cos \phi + 1}{\cos \phi - 1} \right|.$$
these quantities, such that a specific phase shift determines the value of $\Delta_\phi$. A ratio of 1.0 corresponds to $b_2$ and $u_y$ being 90° (or 270°) out of phase, which is in keeping with the idea of a standing radial mode with equal inward and outward propagation. A ratio of 0 implies a phase shift of 180°, consistent with a purely inward radial Poynting vector. The dashed lines represent the observed phase shifts and ratios from the observation (labelled ‘THD’) and the simulation (labelled ‘Sim’). For the observation, these can be obtained from Figure 2, with panel 4 showing a phase shift of 120° during driving, with a ratio estimated from panel 6 of $1$. This is consistent with the predicted values in Figure 13. For the simulation, the ratio and phase shift during driving have been estimated at 0.14 and 140° respectively, using Figures 11 and 12. As before, the relationship between $\phi$ and $\Delta_\phi$ approximates the simulation results well.

With an extremely simplified approach, we have determined the relationship between the radial Poynting vector and the phase shift between $u_y$ and $b_2$. This proves that either can be used as a definitive measure of the end of the driving period: either through a change in the ratio of inward to outward signal of $S_z$, i.e. returning to a standing mode, or through the phase shift returning to 90°. This idea can be extended to considering a more physical model of an inward propagating wave with a smaller amplitude reflected wave, expressed as (in normalized units)

$$u_y = \cos(\omega t + k_x x - k_y y) \cos(k_z z)$$

$$+ R \cos(\omega t - k_x x - k_y y) \cos(k_z z),$$

for $-1 < R < 1$, where $R$ is the amplitude of the reflected wave, $\omega$ the frequency and $k_x$, $k_y$, and $k_z$ the wavenumbers in the $x$, $y$ and $z$ directions respectively. Using equations (1) and (4) with the $z$ dependence stated explicitly, we can calculate $b_2$ given as

$$b_2 = -A \cos(\omega t + k_x x - k_y y) \cos(k_z z)$$

$$+ AR \cos(\omega t - k_x x - k_y y) \cos(k_z z),$$

where

$$A = \frac{\omega}{k_z} - \frac{k_y^2}{\omega k_x},$$

and $\omega^2 = V_0^2 (k_x^2 + k_y^2 + k_z^2)$. For the location of THD, we assume $k_z \approx 0$ by the close proximity to the symmetry line of the driver, and $z = 0$ by the small magnetic latitude ($\sim 3^\circ$). With this equations (8) and (9) become

$$u_x = \cos(\omega t + k_x x) + R \cos(\omega t - k_x x),$$

$$b_z = -A' \cos(\omega t + k_x x) + A' R \cos(\omega t - k_x x),$$

where $A' = k_x / \omega$. To determine the phase shift between these components, we express them as

$$u_x = G \cos(\omega t + \psi),$$

$$b_z = G' \cos(\omega t + \psi'),$$

with

$$G = \sqrt{1 + R^2 + 2R \cos(2k_x x)},$$

$$G' = -A' \sqrt{1 + R^2 - 2R \cos(2k_x x)},$$

$$\psi = \tan^{-1}\left(\frac{1 - R}{1 + R \tan(k_x x)}\right),$$

$$\psi' = \tan^{-1}\left(\frac{1 + R}{1 - R \tan(k_x x)}\right).$$

Hence $u_x$ and $b_z$ can be written as

$$u_x = G \cos(\omega t + \tan^{-1}(\alpha \tan(k_x x))),$$

$$b_z = -G' \cos(\omega t + \pi + \tan^{-1}\left(\frac{1}{\alpha} \tan(k_x x)\right)), $$

where $\alpha = (1 - R)/(1 + R)$ and $-G'$ is positive. The phase is dependent on $R$, $k_x$ and the position in $x$. In order to calculate the phase difference $\phi$ between the components, we consider $b_z$ to be leading, such that the difference is given by

$$\phi = \pi + \tan^{-1}\left(\frac{1}{\alpha} \tan(k_x x)\right) - \tan^{-1}(\alpha \tan(k_x x))$$

$$= \pi + \tan^{-1}\left(\frac{2R}{1 - R^2 \sin(2k_x x)}\right).$$

As in the simplified approach above, we calculate the ratio of inward to outward $S_z$. We firstly calculate $S_x$ using equations (10) and (11) as

$$S_x = u_x b_z,$$

$$= -A' \cos^2(\omega t + k_x x) + R^2 A' \cos^2(\omega t - k_x x).$$

As previously, we seek to express $S_x$ as a single sinusoidal function, which yields

$$S_x = \gamma + C \sin(2\omega t + \delta),$$

where

$$\gamma = R^2 - 1,$$

$$C = \sqrt{R^4 + 1 - 2R^2 \cos(4k_x x)},$$

$$\delta = \tan^{-1}\left(\frac{R^2 - 1}{\tan(2k_x x) (R^2 + 1)}\right),$$

and a constant factor of $A'/2$ has been removed which will not affect further analysis. Hence the ratio of positive to negative signal $\Delta_x$ can be expressed as the maximum outward to the maximum inward Poynting vector as

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig14.png}
\caption{Contour plot of $\phi(k_x x, R)$, the phase by which $b_z$ leads $u_x$, with labelled contours in degrees.}
\end{figure}
\[ \Delta_s = \frac{|R^2 - 1 + \sqrt{R^4 + 1 - 2R^2 \cos(4k_x x)}|}{|R^2 - 1 - \sqrt{R^4 + 1 - 2R^2 \cos(4k_x x)}|}, \] (14)

given that the maximum and minimum of \( S_x \) will occur when \( \sin(2\omega t + \delta) = \pm 1 \).

The functions \( \phi \) and \( \Delta_s \) contain the necessary information to link the phase by which \( b_z \) leads \( u_x \) and the radial Poynting vector ratio. Figure 14 shows a contour plot of \( \phi \) in \( R-k_x \) space, where the value of the reflection coefficient \( R \) is defined over \([-1, 1]\), while \( k_x \) is defined over \([0, \pi]\). This ensures that all possible solutions are considered since \( \phi \) is periodic over \( \pi \) by virtue of the \( \sin(2\omega t) \) term appearing in (12). The contours are labelled in degrees which highlights the symmetry for phase shifts between 90° → 180° and 180° → 270°. These contours allow the values of \( k_x \) to be constrained. For example, considering a phase shift of 120° implies \( 0.6 < |R| < 1 \) which reveals information about the strength of the reflection of the mode. The contours of \( \Delta_s \) have been omitted since \( \phi(k_x, R) \) and \( \Delta_s(k_x, R) \) actually have the same contours in \( R-k_x \) space, despite being on first appearance two completely separate functions (see (12) and (14)). It can be proven whether these contours are in fact the same by considering the gradients of each function. If the gradients are parallel, this implies that the functions share the same contours and hence one can be expressed as a function of the other. In order to show that the gradients are parallel we require

\[ \nabla \phi \times \nabla \Delta_s = 0, \]

\[ = \frac{\partial \phi}{\partial R} \frac{\partial \Delta_s}{\partial (k_x)} - \frac{\partial \phi}{\partial (k_x)} \frac{\partial \Delta_s}{\partial R} = 0. \] (15)

After some algebra, equation (15) can indeed be shown to be satisfied and hence \( \phi = \phi(\Delta_s) \). This implies that each phase shift \( \phi \) corresponds to a precise ratio of outward to inward radial Poynting vector \( \Delta_s \). This is the same conclusion that was determined by the simple analysis of two phase shifted sine waves. In plotting \( \Delta_s \) as a function of \( \phi \) for a fixed \( k_x \) we produce exactly the same plot as in Figure 13. It is, perhaps, surprising that such a plot does not depend upon the choice of \( k_x \). This implies that the relationship between \( \phi \) and \( \Delta_s \) is independent of nodal structure and position. The more rigorous analysis provided here with the addition of a reflected component, provides a better comparison to a more physical situation, but yields the same relationship between the phase shift and the radial Poynting vector as the simple case.

We have shown that the phase shift between \( u_x \) (\( E_y \)) and \( b_z \) is inextricably linked with the overall shape of the radial Poynting vector. The ratio between the positive and negative \( S_x \) signal determines on a continuous scale the phase by which \( b_z \) leads \( u_x \), which ranges from 90° (or 270°) for equally inward and outward, to 180° for purely inward propagation. Figure 13 can be used to determine the validity of an observation as a global mode i.e. if the observed phase shift and ratio of positive to negative radial Poynting vector are a valid pairing. The above analysis also confirms that the change in \( S_x \) from the driven to post driven phases correlates directly with the change in the phase shift. Hence, both the \( S_x \) signal and the phase shift can be used to clearly infer the end of the driving phase. For example in the THD data in Figure 2, we can estimate that the driving stops at \( \sim 06:46 \) UT, where the phase changes to \( \sim 90° \) (panel 4) and the radial Poynting vector recovers to an equally inward and outward signal (panel 6).

6. Summary

The two simulations performed are very similar. In both cases, fast mode waves enter the domain through a disturbance in the compressional magnetic field component \( b_z \). Most of the energy propagates firstly within the azimuthal extent of the driven region shown through \( S_z \), with some energy leaking tailward through \( S_y \). There is no resonance within the domain for either equilibrium as these points would exist beyond the inner boundary. Driving at the second radial harmonic eigenfrequency in each case precludes the appearance of other frequency modes allowing the clear detection of the natural waveguide mode post driving in the compressional components. As mentioned separately in each of the discussion sections, the main influence on signal structure is the satellite location. Here we have investigated two very different signals, that can be explained almost fully by the same simulation just by the positioning of the satellites.

To demonstrate this further, Figure 15 displays the radial and azimuthal Poynting vector components (\( S_z \) and \( S_y \) respectively) plotted against time for four satellite locations in our model waveguide using the parameters from the Cluster simulation. The locations are shown as points A-D in the top panel of the figure, the coordinates of which are: [Figure 15. Top panel shows the positions of 4 satellites placed in the model waveguide with the vertical dashed line indicating the driven region. Bottom four panels display \( S_z \) (black) and \( S_y \) (red) plotted against time at the 4 satellite positions corresponding to those depicted in the top panel.]


A(x = 2, y = 1), B(x = 6, y = 2), C(x = 3, y = 8) and D(x = 6, y = 12), with the z coordinate for all points being close to the magnetic equator at z = 0.45 (all lengths in R$_{E}$). The driver is the same as in the Cluster simulation given in Figures 5 and 6. In each of the plots, $S_y$ is represented by the black line and $S_x$ by the red line. The driven portion of the $x = 10$ R$_{E}$ boundary extends to $y = 5$ R$_{E}$, as shown by the vertical dashed line in the top panel. The vertical dashed line in the bottom four panels demarcates the time when the driver is switched off at $t = 3.98$ minutes. Position $S_y$ corresponds to the locations of the THEMIS satellite THD, and despite using the parameters for the Cluster simulation, the predominant features are the same: an inward radial Poynting vector during driving levelling out to an equally inward and outward signal post driving with a small positive azimuthal Poynting vector during the driven period. The fact that the signals match purely by positioning the satellite at the same point in the guide, shows the importance of the satellite location relative to the driving region close to the boundary as discussed in section 5.3. We see there is energy flow radially inward and outward, with more rays entering radially. This is balanced by one ray exiting the azimuthal boundary.

The azimuthal Poynting vector also changes markedly with the movement of the satellite further downtail. At B, within the driven region, $S_y$ is small compared to $S_x$ and is at times sunward (negative). This occurs due to the position within the driving region, where waves can emanate from the furthest azimuthal extent of this region ($y = 5$ R$_{E}$) and travel sunward. At location C, $S_y$ is more pronounced and purely tailward. This is a feature of the Cluster data caused by the movement of the satellite further downtail. Finally at D, the signal is again purely tailward by nature of the further downtail position. Considering post driving, there is a clear tendency for $S_y$ to indicate a radial standing structure. Outside the driving region there is tailward propagation, whilst inside dispersion leaves only small $k_y$ modes.

In this paper, we have modeled two ULF wave observations from the Cluster (Clausen08) and THEMIS (Hartinger12) satellites using a simple numerical waveguide model. We have developed a new boundary condition at the driven magnetopause that acts as a pressure driver (see Appendix A). The simulation results match favourably to the aforementioned observations and many interesting features may be discerned from the results.

1. The satellite position is of paramount importance in determining the structure of the signal observed. A location tailward of the disturbed region of the magnetopause will result in a purely tailward azimuthal Poynting vector, which explains the stand out feature of the Cluster data. The location of the source region relative to the spacecraft can also be inferred from the Poynting vector components.

2. An overall inward radial Poynting vector signal does not necessarily require coupling to a FLR or a leaky inner boundary to explain the energy loss. The inward energy flow may be entirely balanced by tailward propagation, not necessarily apparent from a perhaps small azimuthal Poynting vector signal. When a net inward energy flow does occur, the point where the signal returns to an equally back and forth oscillation demarcates the time when the driving stops.

3. The phase difference between the radial velocity (azimuthal electric field) and the field aligned magnetic field can be used to infer whether the mode is propagating or standing radially and hence is another indicator of the transition between the driven and post driven phases.

These features of magnetospheric waveguide modes can be used to help interpret observational signatures.

Appendix A: Analysis of Natural Waveguide Frequency with a $b_z$ Driven Boundary

It is of interest to explain the new boundary condition, where the simulation is driven by perturbations in $b_z$ as opposed to the radial displacement $\xi$ or velocity $u_x$ as in previous studies [e.g., Rickard and Wright, 1994; Wright and Rickard, 1995b], from which we can infer the radial nodal structure of the waveguide modes. In these studies, the inner boundary of the waveguide ($x = 0$) is perfectly reflecting with a node of $u_x$, and driving with $u_x$ at the outer boundary ($x = 1$ in dimensionless units), also simulates a node of $u_x$, resulting in a half wavelength fundamental mode. Our aim here is to demonstrate that driving with $b_z$ simulates a node of $b_z$ at the outer boundary, which together with an antinode of $b_z$ at the inner boundary creates a quarter wavelength fundamental radial mode. This has been previously posited by Mann et al., [1999], who suggested that this boundary condition could lower the eigenfrequencies of the waveguide, without resorting to unrealistically large magnetospheric plasma densities.

The driving condition is implemented by overwriting the value of $b_z$ on the $x = 1$ boundary. Throughout the computational domain, centred differences are used for the spatial derivatives, with ghost or halo cells used for the boundary calculations. On the driven boundary however, centred differences cannot be implemented, since no information exists beyond the prescribed boundary value. Hence, for the derivatives in $x$ ($\partial u_x/\partial x$ and $\partial \xi_v/\partial x$), fourth order backwards differencing is employed. In the predictor step of

Figure 16. Waveguide schematic displaying possible ray trajectories.
wavenumbers $k$, eigenfrequencies $\lambda$, wavenumbers $k_2$, and $k_3$, respectively. This results in the radial harmonic matching the boundary conditions are respectively, then there is a quarter wavelength radial structure. If these frequencies can be reproduced from the simulation, it will then be clear that indeed the new boundary condition does enforce a quarter wavelength radial structure.

As a difference to the simulations previously discussed in sections 4 and 5, the $x = 1$ boundary is driven continuously over the full extent in the $y$ direction. The driving frequency is chosen between the first and second harmonic frequencies as $\omega_2 = 4.865$, such that no one frequency is dominantly driven. Figure 17 shows a FFT taken at a point near the middle of the domain in $x$, close to the $y = 0$ boundary. Three clear frequency peaks are visible, matching well to the predicted first and second harmonics and to the driving frequency. A weak response is also observed around the predicted third harmonic frequency. This is clear evidence that driving the outer boundary with the $b_2$ perturbation simulates a node of $b_2$, to give a quarter wavelength fundamental mode.

Acknowledgments. T.E. would like to thank STFC for financial support for a doctoral training grant. Data from simulation results are available on request from T. Elsden, email: t.e55@st-andrews.ac.uk.

References


