Simulations of Alfvén waves in the geomagnetic tail and their auroral signatures

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Abstract. Observations show that Alfvén waves in the magnetotail have distinctive properties depending upon their location. In particular, those in the plasma sheet boundary layer (PSBL) have a larger amplitude and favour earthward propagation compared to those in the tail lobe which have the polarization of standing waves. The PSBL waves are also associated with electron acceleration and optical auroral emissions that exhibit equatorward motion. In this paper we present simulations of MHD wave coupling in the magnetotail to support an explanation for how Alfvén waves with these properties may be established. The simulations also suggest the waves should have periods from 5 mins to >20 mins, and produce auroral emissions in the ionosphere having a latitudinal range of 40-130 km and equatorward speed of order 1 km s\(^{-1}\). Field aligned currents are typically a few \(\mu\)A m\(^{-2}\).

1. Introduction

Four decades ago it was realised that the Earth supported an extended magnetic tail. Early theoretical studies of this structure by McClay and Radoski [1967] and Patel [1968] showed that magnetohydrodynamic (MHD) normal modes of the tail have natural frequencies of the order of (or less than) millihertz, in agreement with magnetometer data [e.g., Herron, 1967]. The study of decoupled fast modes in more realistic tail equilibria has continued to receive attention [e.g., Hopcraft and Smith, 1986; Edwin et al., 1986]. Subsequently the coupling of fast and Alfvén waves in the magnetotail has been of interest [see Liu et al., 1995; Allan and Wright, 1998, 2000; Wright et al., 1999] and has been reviewed in Wright and Mann [2007].

Observations have provided considerable motivation for the above studies. In particular, optical auroral brightenings at the foot points of field lines carrying Alfvén waves have shown a common frequency [Sumson et al., 1996; Xu et al., 1993] with the Alfvén wave fields. This indicates that the electrons carrying the field-aligned Alfvén wave currents are energetic enough to produce auroral enhancements when they precipitate during the upward current phase of the wave cycle. The study of electron dynamics in Alfvén waves is a topic of great interest. For example, Dombeck et al. [2005] show how the Poynting vector decreases with altitude between POLAR and FAST, and is associated with an increased electron energy flux. Similarly Vaivads et al. [2003] show the Poynting vector observed by Cluster when mapped earthward to DMSP is comparable to the electron energy flux there. These observations indicate that the energy required to accelerate electrons to carry the Alfvén wave field aligned current can be a significant sink of wave energy. Indeed Wright et al. [2003] showed this loss mechanism could exceed the traditional damping process associated with Ohmic heating in the ionosphere.

The optical auroral features associated with Alfvénic electron precipitation will share the same latitudinal phase motion as the Alfvén wave. Observations by Liu et al. [1995] and Wright et al. [1999] show equatorward phase motion suggesting the waves are on field lines threading the plasma sheet boundary layer (PSBL). It has been suggested that these waves can account for the Poleward Boundary Intensifications (PBIs) reported by Lyons et al. [2002] (located on the poleward edge of the auroral oval, and thought to map to the PSBL), which are associated with activity in the tail.

The most recent observations in this area are in situ measurements of wave fields in the magnetotail. Lobe Alfvén waves are excited by substorms and can have a standing wave structure, even on open field lines [Keiling et al., 2002]. PSBL Alfvén waves have a larger amplitude than those in the lobe [Wygant et al., 2000; Keiling et al., 2005]. These studies correlate earthward Poynting vector with auroral luminosity, and hence the energy of precipitating electrons, which is sufficient to supply. Interestingly, the PSBL waves appear to be composed of an earthward propagating wave plus a partially reflected (anti-earthward) wave. The latter is sometimes negligible, and ultra-low-frequency (ULF) PSBL waves show a bias towards being earthward propagating and having an earthward directed Poynting vector [Wygant et al., 2000; Keiling et al., 2002, 2005].

In this paper we present simulations that provide an explanation of the following observations:

1. The Alfvén wave amplitude in the PSBL exceeds that in the lobe.
2. The lobe Alfvén waves have a standing structure, whilst those in the PSBL are earthward propagating.
3. The PSBL waves produce optical auroral emissions, whilst those in the lobe do not.
2. Model

2.1. Equilibrium

We model the equilibrium magnetotail as a simple wave-guide. A uniform magnetic field is directed earthward in the northern half (\(B = (B_0, 0, 0), z > 0\)), and anti-earthward in the southern half (\(B = (-B_0, 0, 0), z < 0\)), \(x = 0\) is located in the magnetotail, and \(x\) increases towards the earth. The cross-tail coordinate is \(y\), and completes the right-handed coordinate system. The plasma density is solely a function of \(z\), as is the Alfvén speed, and is taken to be

\[
V_A(z) = \begin{cases} 
V_2 + c_2(z - z_2)^3 & z > z_2 \\
V_2 & z_1 < z < z_2 \\
V_2 - c_1(z_1 - z)^3 & z/2 < z < z_1 \\
V_1 + c_1 z^3 & 0 < z < z_1/2 
\end{cases}
\]

The variation represents a plasma sheet Alfvén speed of \(V_1\) which increases over the PSBL to a lobe value of \(V_2(z_1 < z < z_2)\). There is also a decrease in Alfvén speed throughout the mantle (\(z > z_2\)). The constants \(c_1\) and \(c_2\) are \(c_1 = \frac{1}{3}(V_2 - V_1)(z/z_1)^3\) and \(c_2 = (V_3 - V_2)/(1 - z_2)^3\). These are chosen so that \(V_A(z)\) is a continuous and differentiable function. \(z\) has been normalised by \(z_M\) (the half-width of the tail in the north-south direction) so \(z = 1\) represents the northern edge of the tail mantle region, where \(V_A = V_3\).

The parameters defining the tail equilibrium are \(z_1, z_2, V_1, V_2\) and \(V_3\), and can be chosen to investigate the behaviour of waves in a variety of magnetotail configurations. We normalize all Alfvén speeds to the lobe Alfvén speed \((V_{AL})\) and employ three equilibria in this paper. All equilibria have

\[
V_2 = 1.0, V_3 = 0.5 \text{ and } z_2 = 0.8.
\]

Other parameters are:

- EQU1: \(V_1 = 0.25, z_1 = 0.2\)
- EQU2: \(V_1 = 0.25, z_1 = 0.3\)
- EQU3: \(V_1 = 0.20, z_1 = 0.3\)

Figure 1 displays the variation of \(V_A\) with \(z\) for these three equilibria.

2.2. Governing Equations

As mentioned previously, length is normalized to the tail half-width (in \(z\), \(z_M\)), and velocity by the maximum Alfvén speed in the lobe, \(V_{AL}\). Thus the normalizing time unit is \(z_M/V_{AL}\). Magnetic fields are normalized by the equilibrium field strength, \(B_0\), and hence densities by \(B_0^2/\mu_0 V_{AL}^2\). The normalized linear cold plasma ideal MHD equations for the perturbed magnetic field, \(b = (b_x, b_y, b_z)\), and velocity, \(u = (u_x, u_y, u_z)\) are

\[
\begin{align*}
\partial b_x/\partial t &= -(\partial u_x/\partial z + k_y u_y) \\
\partial b_y/\partial t &= \partial u_y/\partial x \\
\partial b_z/\partial t &= \partial u_z/\partial x + k_y b_x/ho \\
\partial u_x/\partial t &= (\partial b_x/\partial x - \partial b_y/\partial z)/\rho.
\end{align*}
\]

The velocity component \(u_x\) is chosen to have a separable dependence of \(\sin(k_y y)\), and other perturbations have a \(\sin(k_y y)\) or \(\cos(k_y y)\) dependence consistent with this. The above equations then give the evolution of \(b(x, z, t)\) and \(u(x, z, t)\).

2.3. Numerical details

The governing equations are advanced in time by the Leap-frog Trapezoidal scheme: letting \(U = (b_x, b_y, b_z, u_x, u_y, u_z)\) and the components of \(F\) be the r.h.s of (2)-(6), the equations are equivalent to

\[
\partial U/\partial t = F(U).
\]

If \(U\) is known at times \(t\) and \(t - \Delta t\) (where \(\Delta t\) is the integration time step) then \(U^1\) is a prediction for \(U^{t + \Delta t}\) based upon \(U^{t - \Delta t}\) and \(F^0 \equiv F(U^0)\),

\[
U^1 = U^{t - \Delta t} + 2\Delta t F^0. \tag{8}
\]

\(F^1 \equiv F(U^1)\) is thus an estimate of \((\partial U/\partial t)^{t + \Delta t}\), and so \(F^*\)

\[
F^* = (F^1 + F^0)/2 \tag{9}
\]
gives \((\partial U/\partial t)^{t + \Delta t/2}\). Finally we take a centred time integration to find a better estimate for \(U^{t + \Delta t}\),

\[
U^{t + \Delta t} = U^t + \Delta t F^*. \tag{10}
\]

All time integrations are centred and so second order accurate. The steps in (9) and (10) are used to damp the computational mode inherent in the leapfrog step (8).

Figure 1. The variation of Alfvén speed with \(z\) across the northern half of the tail. Length is normalised by the tail half-width, with \(z = 0\) being at the centre of the plasma sheet.
2.3.1. Boundary Conditions. The boundary at \( z = z_M \) is taken to be perfectly reflecting by setting \( u_z = 0 \) there. However, our solutions are fairly insensitive to this condition since the waves we are interested in are trapped in the plasma sheet and PSBL. At the “ionospheric” end \( (z = x_M) \) we represent a perfectly reflecting ionosphere by requiring \( u_z = 0 \) there. To represent a perfectly absorbing ionosphere at \( x_I \) we adopt an “outgoing wave” condition there. This was achieved by setting the grid length in \( x(x_M) \) to be sufficiently greater than \( x_I \) that no waves reflected from \( x_M \) would return to \( x_I \) during the simulation. Hence the waves at \( x_I \) will all be earthward propagating (i.e. propagating “out” of the domain \( 0 < x < x_I \)). The tail boundary of the simulation domain employs the symmetry condition \( \partial u_x / \partial x = 0 \). Other wavefields at the boundaries have nodes/antinodes as required for consistency with the equations (2)-(6).

For computational efficiency we solve only in the space \( z \geq 0 \), and the boundary condition at \( z = 0 \) is chosen to represent either even modes (in \( z \)) of the waveguide \( (u_z = 0) \) or odd modes \( (b_z = 0) \). These conditions are applied along the \( x \) axis at all times except over \( 0 < x < x_d \) during the “driving” phase, \( 0 < t < t_d \), when either \( u_x \) (even) or \( b_z \) (odd) is proportional to

\[
\cos(\pi t/t_d) [1 - \cos(2\pi t/t_d)]^2 [1 + \cos(\pi x/x_d)].
\]

Other variables are updated on \( z = 0 \) according to (2)-(6) using one-sided derivatives in \( z \) when required. (This method of driving was also employed by Wright and Rickard [1995].) Numerical results are normalized by having the maximum value of \( u_x(x = 0, z = 0, 0 < t < t_d) \) equal to unity. Allan and Wright [2000] explain how \( x_d \) and \( t_d \) can be chosen to represent the effect of a plasmoid forming and being ejected. In the even mode simulation the plasmoid is perfectly symmetric about \( z = 0 \). If there is some asymmetry odd modes will also be present. The general case is a superposition of both even and odd modes.

2.3.2. Numerical Accuracy. The wave fields that evolve tend to have a small spatial scale in \( z \) where \( dV_A/dz \) is largest, and can reduce as time increases due to phase mixing [e.g., Allan and Wright, 2000]. It is important to ensure that this scale is properly resolved by the grid. The results in this paper have either \( \Delta z = 2.5 \times 10^{-4} \) or \( \Delta z = 5 \times 10^{-4} \). The time step is then determined by the CFL condition and \( \Delta z \). We used \( \Delta t = 10^{-4} \) and \( \Delta t = 2 \times 10^{-4} \), respectively for the two \( \Delta z \) resolutions. The waves do not have a particularly small scale in the \( x \) direction, and \( \Delta x \) was taken as \( 2 \times 10^{-2} \).

To check numerical convergence we compared the \( u_z \) fields calculated with the above resolution against a simulation using double the resolution in space and time [see Allan and Wright, 2000]. For the results presented here the fields had converged to better than 0.05\%. We also compared the Poynting flux energy flow into the domain across the boundary during driving to the volume integrated energy density at the end of the simulation. This showed our ideal simulations conserved energy well as their ratio was at least 0.99995. Preservation of \( \mathbf{\nabla} \cdot \mathbf{b} = 0 \) was checked, and reached a maximum value of \( 10^{-11} \) throughout the simulations, being limited by machine precision.

![Figure 2](image2.png)  
**Figure 2.** A source of fast modes waves exists at the centre of the plasma sheet, and may be viewed as a set of wavepackets. The behaviour of one particular wavepacket is shown: It has a turning point at \( z_A \) and propagates earthward with a group velocity \( V_g \). A little beyond \( z_A \) (at \( z_I \)) its parallel phase speed matches that of Alfvén waves, and mode coupling occurs. The Alfvén wave travels earthward at speed \( V_A(z) > V_g \), and so runs ahead of the fast wavepacket. (The earth is to the right of this figure.)

![Figure 3](image3.png)  
**Figure 3.** The variation of Alfvén wave amplitude with \( x \) and \( z \) at \( t = 6.0 \). Only the northern half of the tail is shown. The wave source was centred on \( (0,0) \), and energy propagates earthward to \( z > 0 \). The dawn-dusk wavenumber, \( k_y \), is taken to be 1.3 in panel (a) and 7.5 in panel (b). The PSBL \( (0.1 < z < 0.2) \) and lobe \( (0.2 < z < 0.8) \) both carry Alfvén waves with similar amplitude in (a), whilst in (b) those in the PSBL are far larger than those in the lobe. [Model used is EQU1 with \( t_d = 1.0, x_d = 0.48, t = 6.0 \).]

![Figure 4](image4.png)  
**Figure 4.** The variation of Alfvén wave \( b_y \) amplitude across the tail for different \( k_y \). The PSBL is centred on \( z = 0.125 \).
3. Magnetotail Alfvén Waves

The driving conditions described in Section 2.3.1 introduce primarily fast mode disturbances into the tail. If \( k_0 = 0 \) the energy remains in the fast mode whilst dispersing and propagating along the tail waveguide. In this case there is no coupling to Alfvén waves, which are characterized by the \( u_y \) and \( b_y \) fields. When \( k_y \neq 0 \) the fast mode waves will couple to Alfvén waves as shown in Figure 2: The source of waves at \( (x, z) = (0, 0) \) can be thought of as a superposition of fast mode wavepackets, described via the fast mode dispersion relation \( \omega_b(k) \). Here \( k_0 \equiv k_z \), the field-aligned wavenumber, and \( k_y \) has some chosen (fixed) value. The subscript \( n \) refers to the harmonic number in \( z \). On field lines where the Alfvén speed \( (V_A(z)) \) is equal to the field-aligned fast mode phase speed \( (\omega_b(k_y)/k_y) \) efficient mode conversion to the Alfvén mode may take place [e.g., Allan and Wright, 2000], driving an Alfvén wave of frequency \( \omega_A(z) = \omega_b(k_y) \) and wavenumber \( k_{1A}(z) = k_y \). Consideration of a different \( k_y \) identifies a different phase speed \( (\omega_b(k_y)/k_y) \) and hence field line where \( V_A(z) \) matches. Thus \( \omega_A \) and \( k_{1A} \) will vary across the PSBL and lobe. The Alfvén waves run along the field line at a speed \( V_A(z) \), whilst the fast mode wavepacket driving these waves travels along the guide at a slower speed \( V_{\parallel}(k_y) = \partial \omega_b(k_y)/\partial k_y \) (see Wright et al. [1999] and Wright and Mann [2007] for more details).

Allan and Wright [1998] presented the first study of coupled waves in the magnetotail waveguide. To facilitate the interpretation of their simulation results, they adopted a small value for \( k_y \) of 0.5, which is the weak coupling limit. This permitted the use of decoupled \( (k_y = 0) \) modes and dispersion relations as an approximation for the weakly coupled modes, and allowed for a clear identification of the physics operating. Subsequently, Allan and Wright [2000] attempted a realistic study of waves in the tail by adopting a realistic \( V_A(z) \) profile and taking \( k_y = 1.3 \), which corresponds to the fundamental mode from dawn to dusk. For EQU1 (“Model A” in their study) and driving \( u_x \) with parameters \( t_d = 1.0 \) and \( x_d = 0.48 \), Figure 3a shows contours of the Alfvén wave amplitude (when \( k_y = 1.3 \) and \( t = 6.0 \)) through the quantity \( \sqrt{E_A} \), where the Alfvén wave energy density is

\[
E_A = \frac{1}{2}(\rho u_x^2 + b_y^2), \tag{12}
\]

![Figure 5. Comparison of Alfvén wave \( b_y \) amplitude in the PSBL and in the lobe as a function of \( k_y \).](image)

and is adapted from their Figure 4. Notice how the PSBL Alfvén waves (centred on \( z = 0.125 \)) have a strong phase mixing gradient which leads to substantial field aligned currents \( (j_1) \). In contrast the lobe \( (0.2 < z < 0.8) \) has essentially a plane propagating Alfvén wave over \( 3 < x < 6 \) and little phase mixing or \( j_y \).

Whilst these results may agree with many features in observations, they do not account for the recent bias reported by Wygant et al. [2000] and Keiling et al. [2002] for the Alfvén wave \( b_y \) to be greater in the PSBL than in the lobe. In an effort to address this we present new results in Figure 3(b). These have the same parameters as the simulation in Figure 3(a) except \( k_y \) was increased from 1.3 to 7.5. The contour values in the two panels are the same, and it is evident that the Alfvén wave amplitude in the PSBL far exceeds that in the lobe over \( 3 < x < 6 \).

To investigate the wave amplitude dependence on \( k_y \) we performed simulations for several values of \( k_y \). For each run the final snapshot at \( t = 6.0 \) was examined as follows: For a given \( z \) the “Alfvénic” region was identified (typically \( 3 < x < 6 \)) and the maximum amplitude of \( b_y \) logged. This procedure was repeated for all \( z \), and the results are displayed in Figure 4. It is clear that for all but the smallest \( k_y \) there is stronger coupling to Alfvén waves in the PSBL (centred on \( z = 0.125 \)) than in the lobe \( (0.2 < z < 0.8) \). Figure 5 summarizes this behaviour by taking \( b_y(z = 0.125) \) and \( b_y(z = 0.5) \) to represent the wave amplitude in the PSBL and lobe, respectively, and showing their variation with \( k_y \). For a large range of \( k_y \) (>5) the PSBL amplitude exceeds that in the lobe by an order of magnitude or

![Figure 6. Upward field aligned current density at the ionospheric end using EQU2 with \( t_d = 1.0 \) and \( k_y = 7.5 \) as a function of \( x \) and \( z \). Other parameters are given in each panel. The region \( 0.1 < z < 0.2 \) corresponds to the PSBL. The contours are chosen so that, when mapped to the ionosphere, they correspond to upward current densities of 0.5, 1.0, 1.5,... \( \mu \text{A} \text{m}^{-2} \). The driver symmetry only excites modes that are antisymmetric in \( u_x \) about \( z = 0 \).](image)
more. The dependence of coupling strength on wavenumber is familiar from the related, but different, studies in a box model applicable to closed field lines [Kivelson and Southwood, 1986].

4. Electron Acceleration and Auroral Signatures

Observations show that the large Poynting vector in the PSBL is correlated with auroral intensity [Wygant et al., 2000; Keiling et al., 2002] and hence energy of precipitating electrons. Auroral enhancements on the poleward edge of the auroral oval are thought to map to the PSBL and show repetitive equatorwards motion [Liu et al., 1995; Allan and Wright, 1998; 2000]. Observations suggest that much of the earthward Poynting flux can be converted to precipitating electron energy flux [Wygant et al., 2000], which could lead to a significant loss of wave energy [Wright et al., 2003] and an absence of a reflected wave from the ionosphere. This would be expected to occur predominantly on PSBL, rather than lobe, field lines since \( j_y \propto \partial b_y / \partial z \): PSBL field lines not only have a relatively large \( b_y \), but also strong phasemixing gradients which produce large \( j_y \). As the waves propagate into the converging field geometry near the earth, \( j_y \) intensifies even further requiring substantial electron acceleration to keV energies to carry the current. In the lobe, the smaller wave amplitude and absence of phasemixing means that only weak \( j_y \) exists there, and electron motion does not reduce the amplitude of the wave reflected from the earth nor produce auroral displays of the same intensity as on PSBL field lines. These properties suggest efficient reflection of lobe Alfvén waves will lead to a superposition of earthward (incident) and anti-earthward (reflected) waves. As seen in the simulations of Allan and Wright [2000] the combination gives the polarization of a local standing wave, as also seen in data recorded in the lobe [Keiling et al., 2005]. In contrast, the smaller reflection coefficient appropriate for PSBL field lines suggest a mixed polarization somewhere between a standing wave and an earthward propagating wave is likely, and is also in accord with Keiling et al.’s observations.

4.1. Even Modes

To investigate the likely auroral signatures in optical data we choose the outgoing wave condition at the ionospheric boundary, since the main area of interest is the PSBL field lines. The field-aligned current is calculated from \( \mu_0 j_y = \partial b_y / \partial y - \partial b_y / \partial z \) at the notional ionospheric boundary. In the simulation, this boundary is simply the plane \( x = x_j \) which we then map to give representative ionospheric fields by reducing the \( z \) dimension by a factor of \( \sim 40 \) and increasing \( j_y \) by a factor of \( 10^3 \). (The details of the normalization are discussed fully later, but for the moment we note that the normalizing length is \( z_M = 25 R_E \), and plots of upward current density in the ionosphere have contour values of \( 0.5, 1.0, 1.5, 2.0 \ldots \mu A m^{-2} \).)

Figure 6 shows simulations of even modes (i.e., ones for which \( u_z \) is antisymmetric about \( z = 0 \)). The contours indicate upward ionospheric currents that would be expected to produce optical auroral emissions. The background model is EQU2 (\( V_1 = 0.25, V_2 = 1.0, V_3 = 0.5, z_1 = 0.3, z_2 = 0.8 \)), the duration of the driver was \( t_d = 1.0 \), and the strong coupling limit (\( k_y = 7.5 \)) is assumed. In Figure 6(a) the driving displacement (centred on \( x = 0 \)) has an extent \( x_d = 0.48 \), and the ionosphere is at \( x_1 = 2.0 \). Three equatorward moving arcs are evident, associated with upward currents exceeding \( 1 \mu A m^{-2} \).

To explore the dependence of arc structure on our parameters, we changed the extent of the driver to \( x_d = 0.24 \) and show the results in Figure 6(b). (All other parameters are as in (a).) The effect is to intensify the currents (\( \sim 4 \mu A m^{-2} \)) and increase the latitudinal range of the arcs. The slope of the arcs gives the north-south phase speed which Wright et al. [1999] show is given by

\[
V_{ps}(x, z, t) = \frac{-V_A(z)}{dV_A/dz} \frac{V_A(z) - V_g(z)}{x - V_g(z) t}.
\]

Here \( V_{g//}(z) \) is the parallel group velocity of the fast mode that couples to Alfvén waves on the field line at \( z \). Note that \( V_{ps} \propto (dV_A/dz)^{-1} \), and this accounts for the increasing slope at the top of the PSBL (\( z \approx 0.18 \)) seen clearly in the

![Figure 7.](image-url)
first arc and data [e.g., plate 2 of Wright et al., 1999; Figure 2 of Lyons et al., 2002].

In Figure 6(c) we adopt the same parameters as in (b), except that the ionosphere is closer ($x_I = 1.0$, rather than 2.0). This results in an earlier arrival time of the waves at the ionosphere. We also note that the arcs in (c) have a greater phase velocity (steeper slope) than in (b). This can be understood using (13) to estimate the phase velocity at the leading edge of the Alfvén wave signal at a given $z$: If the waves here are propagating at speed $V_A(z)$, the leading edge will be located at $x = V_A(z)t$. Hence (13) gives

$$V_{px}(x) = \frac{-V_A}{\frac{dV_A}{dz}} \cdot \frac{1}{t}$$

(14)

and it is evident that the earlier the first arc appears, the larger its phase speed will be. This can also be understood physically: As the Alfvén waves propagate earthward they phase mix and develop structure in the $z$ direction. The further they propagate, the smaller the scales in $z$ (and larger $k_z$) becomes. Thus the phase speed ($V_{px} = \omega_A(z)/k_z$) becomes reduced as $x_I$ is increased.

4.2. Odd Modes

The even fast modes of the tail that were considered in the previous subsection represent only half the normal modes. The other half are odd, and have an antinode of $v_z$ at $z = 0$. The fundamental mode is odd and corresponds to a flapping motion of the tail in which the central plasma sheet is displaced from its equilibrium position.

We investigate the behaviour of the odd modes of the tail in equilibrium EQU3 ($V_1 = 0.2, V_2 = 1.0, V_1 = 0.5, z_1 = 0.3, z_2 = 0.8$) and retain $k_y = 7.5$ which still corresponds to efficient coupling to Alfvén waves.

Figure 7 displays the expected auroral features through the proxy of upward current intensity contours. In (a) the driver parameters are $x_d = 0.48$ and $t_d = 1.0$, whilst the ionosphere is at $x_I = 2.0$. Fairly weak currents are produced in the PSBL near where $dV_A/\frac{dz}$ has its maximum value ($z = 0.15$). If the driver is localized more ($x_d = 0.24$) the effect is to raise higher harmonics [as noted by Allan and Wright, 2000], and with a greater amplitude as seen in (b) leading to large $j_y (\sim 3.5 \mu A m^{-2})$. The shortest period signatures appear to be confined to the outer PSBL ($z = 0.15$).

The effect of increasing the driver period from $t_d = 1.0$ (a) to $t_d = 2.0$ (c) is to increase the amplitude of $j_y (\sim 2 \mu A m^{-2})$. The longer driving period also places more energy in the lower frequency fast modes, which couple to corresponding lower frequency Alfvén waves (found at lower $z$).

In panel (d) we see the effect of increasing $x_d$ to 0.72 and increasing $t_d$ to 0.25 (compared to (a)) is to produce arcs of moderate current strength ($j_y \sim 1.5 \mu A m^{-2}$) without higher harmonics present. If these parameters are retained, but the ionosphere is moved from $x = 2.0$ to $x = 3.0$, the results in (e) are produced. Here we can clearly see how the increased propagation time allows for more phase mixing which results in a larger $k_s$, associated with smaller phase speeds and an enhanced $j_y (\sim 2.5 \mu A m^{-2})$.

5. Comparison with Observations

To obtain realistic quantities from our simulations we adopt the following normalization: $B_0 = 10 nT; V_{AL} = 700 \text{ kms}^{-1}$ (lobe Alfvén speed); $z_M = 25 R_E$ (tail half-width). These give our time unit as 227.5 secs (about 4 mins). Allan and Wright [1998; 2000] suggest the driving parameters we adopt are representative of space and time scales associated with plasmoid ejection, when the amplitude of $u_x(0,0,0 < t < t_d)$ is $\approx 320 \text{ kms}^{-1}$.

The cross-tail (dawn-dusk) scale of the waves, when $k_y = 1.3$, gives a half wavelength of 60 $R_E$ and corresponds to the fundamental mode in $y$. This was the value of $k_y$ used in Allan and Wright [2000] and Figure 3(a). The strong coupling of $k_y = 7.5$ employed in Figures 6 and 7 give a half wavelength of 10 $R_E$, and it is likely that plasmoids or wave sources with this extent in $y$ will produce fast modes that couple particularly efficiently to Alfvén waves in the PSBL. The distances to the ionosphere, $x_I = 1.0, 2.0$, and 3.0 correspond to 25, 50 and 75 $R_E$, respectively.

In order to map the solution at $x_I$ on to the ionosphere and allow for the converging field geometry we assume $B$ increases by a factor of $10^3$, and causes $j_y$ to increase by this factor also. With these considerations the upward current contours in Figures 6 and 7 correspond to values $0.5, 1.0, 1.5 \ldots \mu A m^{-2}$ in the ionosphere. When mapping field lines to the ionosphere, their separation decreases by a factor of around 40. This means a $z$ interval of 0.1 in Figures 6 and 7 is equal to 2.5 $R_E$ in the tail, but maps to 400 km (3.6° of latitude) in the ionosphere.

The north-south extent of the $>0.5 \mu A m^{-2}$ electron precipitation for the even modes (Figure 6), when mapped to the ionosphere corresponds to 40-80 km, and is similar for the odd modes in Figure 7 (35-130 km). This compares favourably with the data reported by Wright et al. [1999] (120-200 km) and Lyons [2002] (60-200 km) given the crude mapping we employ.

The period of the arcs produced by even modes (Figure 6) is 4.5-6 min. This is similar to the higher harmonic arcs associated with odd modes (Figure 7(b)). The longest periods result from the fundamental odd mode and vary with driving parameters and location in the PSBL. They are typically 10-15 min, but the longest exceed 20 min. These agree well with the periods reported by Wright et al. [1999] of 5.4, 9.8, 16.7 and 18.5 min, and those of Lyons et al. [2002] (13-15 min and 25-30 min).

The phase speeds in the ionosphere of the arcs in Figure 6 range from 0.28 to 0.72 kms$^{-1}$, while those in Figure 7 span 0.17 to 0.4 kms$^{-1}$. These compare well with the observations in Wright et al. [1999] of 0.34-0.76 kms$^{-1}$, and 0.5-1.0 kms$^{-1}$ [estimated from Figure 2 of Lyons et al., 2002].

Observations show that the Alfvén wave magnetic field is larger in the PSBL than in the lobe [Wygant et al., 2000; Keiling et al., 2002], and that the Poynting vector in the PSBL exceeds that in the lobe by 2-3 orders of magnitude [Keiling et al., 2005].

Our simulations (see Figure 4) suggest that the waves will need to have a dawn-dusk wavenumber of $k_y \approx 7.5$ (dimensional half-wavelength of 10 $R_E$), and give $b_y$(PSBL) exceeding $b_y$(lobe) by a factor of 20. The ratio of the Poynting vector in the PSBL and lobe may be expressed as

$$\frac{(b_y u_y B_0 / \mu_0)_{PSBL}}{(b_y u_y B_0 / \mu_0)_{lobe}} = \left(\frac{b_y^2 V_A}{b_y^2 V_A}\right)_{PSBL}$$

(15)

Given that $u_y B_0 = \pm b_y V_A$, and that $V_A$ in the center of the PSBL is about 0.5 of that in the lobe (Figure 1), the Poynting vector ratio is $\sim 200$, in accord with Keiling et al.’s [2005] estimates. The scenario we advocate assumes
that the larger Poynting vector in the PSBL is necessary to accelerate electrons to carry the intense $j_3$ that exists there. As these electrons precipitate and produce auroral intensification, it provides a natural explanation for the correlation of Poynting vector with auroral intensity [Wygant et al., 2000; Keiling et al., 2002] and hence energy of precipitating electrons. The significant conversion of Poynting flux associated with the earthward propagating PSBL Alfvén wave to electron energy flux leads to the absence of a strong reflected wave from the near-earth region, in contrast to the behaviour on lobe field lines where the superposition of waves gives a standing wave polarization. These features are seen in the spacecraft data reported by Keiling et al. [2005]. The relation between upward current, Poynting vector, and auroral intensity was assumed to allow us to relate the upward current contours in Figures 6 and 7 to auroral arc intensity data.

6. Summary

In this paper we have performed numerical simulations of MHD wave coupling in a magnetotail waveguide. The properties of the Alfvén waves that are produced agree with many features seen in data. The picture that emerges is shown schematically in Figure 8.

1. Energy release in the tail such as from substorms can release fast mode energy into the magnetotail waveguide.

2. Earthward of the site of energy release the fast modes couple to earthward propagating Alfvén waves. Those in the lobe travel the fastest, and their arrival at the earth can be used as an indicator of substorm onset [Keiling et al., 2005]. If the dusk-dawn extent of the energy release site is $\sim 10R_E$, the PSBL Alfvén waves will have a much larger amplitude and Poynting vector than those in the lobe.

3. The nonuniform Alfvén speed in the tail refracts the Alfvén waves and they develop phase structure in the $z$ direction through the process of phase mixing. This occurs most efficiently where $dV_A/dz$ is largest. Hence the wavelength in $z$ is much smaller in the PSBL than in the lobe.

4. The Alfvén wave $j_1$ is proportional to the wave’s $b_y$ amplitude divided by the wavelength in the $z$ direction. Both of these factors contribute to $j_{23}$ in the PSBL far exceeding that in the lobe.

5. $j_1$ is carried predominantly by electrons travelling along field lines. The speed of electrons in the PSBL, and hence the energy transferred to them from the Alfvén wave, is greater than that in the lobe. This behaviour is enhanced as the current flow approaches the earth, where the converging field geometry intensifies $j_3$ further.

6. The downgoing electrons precipitate in the ionosphere. On PSBL field lines $j_3$ can reach several $\mu$A/m$^2$, requiring electrons to have energies of several keV. Much lower energies are expected on lobe field lines. PSBL precipitating electrons will have sufficient energy to produce optical auroral emissions, whilst those on lobe field lines will generally not. The phase motion of the arcs on PSBL field lines will be the same as that of the Alfvén wave; namely, equatorward with speeds of about 1 km/s.

7. The energy given by PSBL Alfvén waves to electrons may be so significant that there is little or no reflected wave here from the near earth and ionospheric environment. This means PSBL waves will favor the polarization of earthward propagating waves, with a Poynting vector directed earthward. In contrast, lobe Alfvén waves lose little energy to electrons and may undergo inefficient reflection from the ionosphere. This will result in a superposition of earthward and anti-earthward propagating Alfvén waves in the lobe, which will have a polarization similar to a standing Alfvén wave and a Poynting vector whose direction alternates throughout a wave cycle.

Acknowledgments

References


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