Small Scale Structures in Auroral Field-Aligned Currents

Andrew N. Wright

University of St Andrews

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Upward and Downward Currents and the Auroral Acceleration Region

- **Upward Current:**
  - Magnetospheric electrons precipitated \(\Rightarrow\) visible aurora

- **Downward Current:**
  - Ionospheric electrons evacuated to magnetosphere

[Marklund et al., *Nature*, 2001.]
Cluster Observations of Alfvén Waves


- (1) 200–300 s:
  - Quasi-steady, moving over satellites
- (2) 300–380 s:
  - Downward Alfvén wave, no time shift, temporal fluctuations at 20–40 s period, $\lambda_\perp \sim 120 \text{ km, } 4 R_E$
- (3) 380–445 s:
  - Quasi-steady, moving over satellites

[Karlsson et al., 2004.]
Cluster Observations of Particles

- 100–200 s:
  - Upward current

- 200–290 s:
  - Downward current

- (3) >300 s:
  - Mixture of up/downward currents

- Outer layers: quasi-static downward FAC sheets with upgoing electron beams

[Johansson et al., 2004.]
Modeling of Field–Aligned Currents

- Nested quasi-steady Alfvén current sheets
  - Upward and downward currents
- Downward current region depletes ionosphere
  - Ionospheric Alfvén resonator produces small scales
  - Downward current channel broadens

[Streltsov and Karlsson, 2008.]
Development of Small Scales

- \( j_\parallel \) spike appears at boundary of upward and downward currents
- \( j_\parallel = - (E_\perp \cdot \nabla) \Sigma_p - \Sigma_p (\nabla \cdot E_\perp) \)

[Streltsov and Lotko, 2004.]
Ionospheric Depletion Observations [Aikio et al., 2004]

[From Streltsov and Lotko, 2008.]

- Left: All-Sky camera image of discrete auroral arc
- 6\textsuperscript{th} February 2001, around 23:36 UT
- EISCAT(post–midnight)=\(\nabla\)

- Right: EISCAT \(n_e\) variation with altitude and time
- At 150 km altitude \(n_e\) drops from \(10^{11}\) m\(^{-3}\) to \(10^{10}\) m\(^{-3}\) over \(\sim 1\) min.
Estimate of E-region Depletion Time

- Electron content of nightside E-region \((n = 5 \times 10^{10} \text{ m}^{-3}, \Delta h = 20 \text{ km})\)

\[
N \approx n \times \Delta h \approx 10^{15} \text{ m}^{-2}
\]

- Outgoing electron flux through downward current \((j_{\parallel} \approx 5 \mu\text{Am}^{-2})\)

\[
v_{\parallel} = -\frac{j_{\parallel}}{e} \approx 3 \times 10^{13} \text{ m}^{-2}\text{s}^{-1}
\]

- Depletion time \((t_d)\)

\[
t_d \approx \frac{N}{nv_{\parallel}} \approx 30 \text{ s}
\]

Crude estimate: Sources are neglected

- Other studies
  - Karlsson and Marklund [1998], \(10 \mu\text{Am}^{-2} \Rightarrow t_d \sim \text{few s}\)
  - Doe et al. [1995], \(0.02 - 0.2 \mu\text{Am}^{-2} \Rightarrow t_d \sim 30 - 60\text{s}\)
  - Blixt and Brekke [1996]
Downward Current Alfvén Wave Interaction with the Ionosphere

- Plasma velocity in magnetosphere
- Field lines are dragged and tilted
- Field lines slip (diffuse) through ionosphere

- Velocity shear causes magnetic field shear ($j_\parallel$)
- Downward $j_\parallel$ evacuates electrons
- Pedersen current removes ions to maintain quasi-neutrality
Governing Equations I

- Ionospheric electron continuity equation
  \[
  \frac{\partial n}{\partial t} = \frac{1}{e} \frac{\partial j_z}{\partial z} + \alpha (n_e^2 - n^2) \Rightarrow \frac{\partial N}{\partial t} - \frac{j_z(y, z = h, t)}{e} = \frac{\alpha}{h} (N_e^2 - N^2)
  \]
  \((n_e \text{ and } N_e \text{ are equilibrium values.})\)
- Alfvén wave reflection coefficient (velocity) is
  \[
  r = \frac{1 - \mu_0 \Sigma_p V_A}{1 + \mu_0 \Sigma_p V_A}
  \]
  \(\Sigma_p = \text{height integrated Pedersen conductivity}; \text{ Alfvén speed } V_A = B_0 / \sqrt{\mu_0 \rho}\)
- The removal of electrons reduces \(\Sigma_p\)
  \[
  \Sigma_p = \Sigma_{p0} \bar{N} \ ; \quad \bar{N} = \frac{N}{N_e}
  \]
  thus modifying the reflection coefficient
Governing Equations II

- Ionosphere–Magnetosphere boundary condition on total Alfvén wave fields

\[ b_x^T = \mu_0 \sum_p u_x^T B_0 \]

- Combining with Ampère’s Law gives [Cran–McGreehin et al., 2007]

\[
\frac{\partial \bar{N}}{\partial \bar{t}} + \frac{\partial}{\partial \bar{y}} \left( \frac{\eta \bar{u}_i \bar{N}}{1 + \beta \bar{N}} \right) = \bar{\alpha} (1 - \bar{N}^2)
\]

- Normalized quantities;

\[
\bar{y} = \frac{y}{y_0}; \quad \bar{u}_i = \frac{u_i}{u_{i0}}; \quad \bar{t} = \frac{t}{\tau} = \frac{tu_{i0}}{y_0}; \quad \bar{\alpha} = \frac{\alpha y_0 N_e}{u_{i0} \bar{h}}; \quad \eta = \frac{2 \Sigma p_0 B_0}{N_e e}; \quad \beta = \sqrt{\frac{\mu_0}{\rho_0} \Sigma p_0 B_0}
\]

- Typical values: \( \Sigma_{p0} = 1 \) mho, \( B_0 = 5 \times 10^4 \) nT, \( n_e = 5 \times 10^{10} \) m\(^{-3}\), \( h = 20 \) km, \( N_e = 10^{15} \) m\(^{-3}\), \( \alpha = 3 \times 10^{-13} \) m\(^3\)s\(^{-1}\), \( \beta = 1370 \), \( \eta = 3.12 \)
Numerical Solutions

- Snapshots of $\vec{N}(\vec{y}, \vec{t})$ for different incident $j_{\parallel 0}$
- In terms of typical ionospheric current density, $j_{\parallel 0}$ is
  (a) $2.0 \ \mu\text{Am}^{-2}$; (b) $3.0 \ \mu\text{Am}^{-2}$; (c) $8.0 \ \mu\text{Am}^{-2}$; (d) $15.0 \ \mu\text{Am}^{-2}$;
Downward Current Channel Properties

Approximate analytical results [Cran–McGreehin et al., 2007]:

- Behaviour characterised by dimensionless quantity $\mathcal{W} = j_0 \| / (\alpha e n_e^2 h)$
- $\mathcal{W} < 1$:
  - E–region partially evacuated
  - current channel maintains initial width $y_0$
  - timescale of depletion $t_d = 1 / (2\alpha n_e\sqrt{1 - \mathcal{W}})$
- $\mathcal{W} > 1$:
  - E–region completely evacuated
  - current channel broadens to width $y_0 j_0 \| / (\alpha e n_e^2 h)$
  - timescale of broadening $t_d = 1 / (\alpha n_e)$
  - channel broadens to access enough electron production to carry the imposed current
Cluster Observations of Downward Current Broadening

- Nightside auroral oval (3.6 h MLT, 4.3 $R_E$, 69.8° mag lat)
- String of pearls configuration
- Data time shifted for simultaneous encounter
- Diverging electric fields and downward current observed.
- Mapped to ionosphere;
  - Channel width increases from 15 km to 50 km over 280 s
  - $j_{||0} = 15 \, \mu\text{Am}^{-2}$
  - Agrees with modelling
    ($n_e = 6.88 \times 10^{10} \, \text{m}^{-3}, \, h = 20 \, \text{km}, \, \alpha = 3 \times 10^{-13} \, \text{m}^3\text{s}^{-1}$)
All-Sky Camera Observations of Black Stripe Broadening

Michell et al., GRL, 2008

Midnight auroral active region

Downward current (black stripe) embedded between upward currents

Top: Snapshots of Intensity vs distance along N-S cut

Bottom: Intensity(distance, time) along cut

Stripe broadens in time
Channel broadens from 15 km to 50 km over 50 s

Downward current density $\approx 50 \text{ to } 75 \mu\text{A} \text{m}^{-2}$
Small Scales with no Ionospheric Alfvén Resonator

- Magnetospheric Alfvén speed constant
- No Ionospheric Alfvén Resonator
- Small scales are still generated
- What is the physics of small scale generation?

[From Streltsov, private communication, 2008.]
Comparison of Simulations for Upward–Downward Current Pair


[From Russell, private communication, 2009. Uniform B.]
Numerical Scheme Details

\[ \frac{\partial \tilde{N}}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{y}} \left( \eta \tilde{u}_i \tilde{N} \frac{1 + \beta \tilde{N}}{1 + \beta \tilde{N}} \right) = \tilde{\alpha}(1 - \tilde{N}^2) \]

- Simple finite difference scheme: 1st order in time

- Spatial derivatives
  - do not use centred differences
  - left propagating discontinuity requires one-sided (forward) differencing

- Testing the code
  - if \( \tilde{\alpha} = 0 \), then \( \int \tilde{N} d\tilde{y} \) is conserved
    \( (\beta = 12.2, \eta = 1.015, \delta \tilde{y} = 0.01, \delta \tilde{t} = 0.001 \) gives conservation to \( < 10^{-14} \).)
  - convergence to steady state:
    \[
    \int_{-\infty}^{+\infty} (1 - \tilde{N}^2) d\tilde{y} = 0
    \]
    Integral typically \( < 10^{-11} \) for large times

- Reproduces Cran-McGreehin [2007] results
Downward Current Channel Properties

- Behaviour characterised by dimensionless quantity $\mathcal{W} = j_{\|0}/(\alpha e n_e^2 h)$

- $\mathcal{W}$ represents ratio of
  - downward current electron sink $(j_{\|0}/e)$
  - equilibrium production rate $(\alpha n_e^2 h)$

- For the previous results $\mathcal{W} = 1.45$
  - expect slight downward current broadening
  - expect discontinuity to appear in $N$ evolution

- see movie multi_beta100.mov (Russell, 2009)
- see movie for different $\beta = \mu_o \Sigma_{p0} V_A$, $\mathcal{W} = 1.45$ 4betas.mov (Russell, 2009): [Blue = 6.1, Green = 12.2, Orange = 18.3, Red = 100.]
Steady State Results for different $\beta = \mu_0 \Sigma_{p0} V_A$

- $\mathcal{W} = 1.45$: Discontinuity during evolution
- $\beta = \mu_0 \Sigma_{p0} V_A$: Blue = 6.1, Green = 12.2, Orange = 18.3, Red = 100.
- Small scales present in steady state: $\Delta y \propto 1/\beta$
Red—Simulation; Green—Analytical approximation

Time evolution in movie an3b_beta100.mov

Min $\bar{N}$ at $\bar{y}_*$ where $\bar{N}(\bar{y}_*) = 1/[\beta (\mathcal{W} \times [d\bar{u}_i/d\bar{y}]_{\bar{y}_*} - 1)]$, if $\mathcal{W} > 1$

Balance between $j_{\parallel} = -(E_{\perp} \cdot \nabla)\Sigma_p - \Sigma_p(\nabla \cdot E_{\perp})$: see current_beta100.mov
Concluding Remarks

- If $\mathcal{W} = j_{\|0}/(\alpha e n_e^2 \hbar) > 1$ a discontinuity develops

- Are small scales a result of the “sheet” ionosphere model?

- Discontinuities suggest new physics will become important
  - Electron inertial scale effects: $\lambda_e^2 = m_e/\left(\mu_0 n_e e^2\right)$
  - Finite Larmour radius effects: $\rho_s = c_s/\Omega_p$
  - Modified Alfvén wave dispersion relation

$$\omega^2 = k^2 V_A^2 \left(1 + \frac{\rho_s^2 k^2}{1 + \lambda_e^2 k^2_{\perp}}\right)$$

- Energization of electrons could result in a downward Alfvén wave polarization at the current spike

- The current spike could radiate inertial Alfvén waves with $\lambda_{\perp} \sim 10\lambda_e$