Solar coronal heating by magnetic cancellation: II. disconnected and unequal bipoles

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ABSTRACT

Two-dimensional numerical magnetohydrodynamic simulations of a cancelling magnetic feature (CMF) and the associated coronal X-ray bright point (XBP) are presented. Coronal magnetic reconnection is found to produce the Ohmic heating required for a coronal XBP. During the BP phase, about 90 to 95% of the magnetic flux of the converging magnetic bipole cancels at the base. The last 5 to 10% is cancelled when reconnection occurs at the coronal base. Reconnection happens in a time-dependent way in response to the imposed converging footpoint motions. A potential field model gives a good first approximation to the qualitative behaviour of the system, but the MHD experiments reveal several quantitative differences: for example, the effects of plasma inertia and a pressure build-up in-between the converging bipole are to delay the onset of coronal reconnection and to lower the maximum X-point height.

Key words: magnetohydrodynamics (MHD) – Sun: corona – Sun: magnetic fields – Sun: reconnection

1. INTRODUCTION

Observations have revealed that the temperature of the quiet-Sun corona is maintained at a level of over a million degrees Kelvin, even though at solar minimum there are usually weeks without any large-scale coronal events such as solar flares or coronal mass ejections. Thus, it is likely that the many small-scale events in the quiet-Sun corona (such as bright points, microflares and nanoflares) provide almost all the heat (e.g., Parnell & Jupp 2000, Parnell & Galsgaard 2004). The small-scale coronal magnetic field has a complex structure, where photospheric magnetic fragments of opposite polarity have multiple connections (e.g., Beveridge et al. 2002, 2003, Close et al. 2004, Brown et al. 1999). These magnetic sources are in restless motion, fragmenting and merging, while new flux is continually emerging and old flux is cancelling. Consequently, this should lead to continual widespread magnetic reconnection and subsequent local heating throughout the whole corona, even when the Sun is quiet (Priest et al. 2002, 2003, 2005). Indeed, it has been found recently from observations that the coronal flux recycling time (which is the time it takes to reconnect all the coronal flux) is only at most 1.5 hours in the quiet-Sun corona (Close et al. 2004), which is around 1/10 of the photospheric flux recycling time (Hagenaar 2001).

One type of observed small-scale heating phenomenon, which is associated with small-scale magnetic bipoles, is coronal X-ray bright points (XBPs), which are spread over all latitudes and longitudes of the low quiet-Sun corona (Golub et al. 1974). Coronal XBPs have been observed to release energies ranging from $10^{27}$ to $10^{29}$ erg, and they are likely to account for at least about 20% of the heating of the quiet-Sun corona (see von Rekowski et al. 2005 [hereafter referred to as vRPP05] for more details). These values for the released energies are based on the release rates for the heating from BPs observed by Habbal & Withbroe (1981), between $3 \times 10^{23}$ erg s$^{-1}$ and $10^{26}$ erg s$^{-1}$. The lifetimes of a BP observed by Golub et al. (1976) range between 2 and 48 hours with an average of about 8 hours. The rate of occurrence of XBPs over the whole quiet Sun is $\approx 5 \times 10^4$ per day (if caused by small-scale bipoles of emerging flux, so-called ephemeral regions [ERs]) and $\approx 10^3$ per day (if caused by small-scale bipoles of cancelling flux, so-called cancelling magnetic features [CMFs]; Martin et al. 1985). Taking an energy release of $2 \times 10^{27}$ erg per XBP, this results in a total heating rate from XBPs due to both ERs and CMFs of about $3 \times 10^{32}$ erg per day on the whole quiet Sun, i.e. $\approx 6 \times 10^4$ erg cm$^{-2}$ s$^{-1}$ (see vRPP05 for references). This energy gain rate from XBPs is about 20% of the energy loss rate in the quiet-Sun corona, which Withbroe & Noyes (1977) report to be $3 \times 10^5$ erg cm$^{-2}$ s$^{-1}$.

As mentioned above, around two thirds of coronal XBPs are caused by CMFs (e.g., Harvey 1985). Flux cancellation is the mutual loss of flux in equal proportions from each polarity (plus and minus). In an XBP event, possibly more than one photospheric filament of each polarity is interact-
ing, as can also be the case in a cancellation event. CMF's involve flux convergence (Livio et al. 1985) and flux submergence (Harvey et al. 1999), and significant downflows can be present (Chae et al. 2004). Martin et al. (1985) report observations of total cancellation times of 1 to 24 hours. Cancellation, coronal reconnection and coronal heating through XBP formation are linked, and studying these processes is important for solving the coronal heating problem.

Theoretical models of XBPs associated with CMFs have been developed in two and three dimensions by, e.g., Priest et al. (1994), Parnell et al. (1994a,b), Parnell & Priest (1995), Longcope (1998), Longcope & Kankelborg (1999). Using three-dimensional (3-D) magnetohydrodynamic (MHD) simulations, qualitative numerical studies of XBPs due to converging magnetic flux events have been made by, e.g., Dreher et al. (1997) and Neukirch et al. (1997). Quantitative numerical studies of CMFs and the associated XBPs are presented by vRPP05, using 2-D MHD simulations. Building on the numerical model developed by Rickard & Priest (1994), vRPP05 were able to run their MHD experiments until cancellation was completed, in contrast to Rickard & Priest (1994). The initial magnetic configuration in vRPP05 has bipolar sources that are partially connected in such a way that the opposite-polarity sources marginally touch at the base. For this paper, we have developed a model where we start with a completely disconnected inner bipole, as is the case in the theoretical models of Priest et al. (1994) and Parnell et al. (1994a,b). Thus, our simulations include both a complete CMF event and also a complete associated coronal XBP. Furthermore, we consider also unequal sources where the sources of the inner bipole have the same size and total flux, but a different flux distribution and hence a different maximum field strength. We compare our dynamically computed magnetic field configurations with the potential fields calculated from the converging sources at the base.

In both potential and numerical models, there is an overlying coronal field (which is created by flux connecting two outer sources of opposite polarity), and inner magnetic bipolar sources are moved towards each other by a horizontal flow at the base. This flow mimics footpoint motions and is the driver of the system, triggering cancellation and coronal reconnection and hence also creating the heating.

2 THE MODELS

2.1 Basic equations

We model a part of the quiet-Sun corona (with the base located in the low corona), which we assume to be isothermal and where we neglect gravity. Using Cartesian coordinates, we solve the time-dependent MHD equations in two spacial dimensions \((x, z)\), namely, the continuity equation,

\[
\frac{\partial \ln \rho}{\partial t} + \bm{u} \cdot \nabla \ln \rho = -\nabla \cdot \bm{u},
\]

the Navier–Stokes equation,

\[
\frac{\partial \bm{u}}{\partial t} + (\bm{u} \cdot \nabla)\bm{u} = -\frac{1}{\rho} (\sigma \nabla \rho + \Delta A_y \nabla A_y / \mu_0 - \nabla \cdot (\mu_0 \nabla \rho)),
\]

and the uncurred induction equation,

\[
\frac{\partial A_y}{\partial t} + \bm{u} \cdot \nabla A_y = \eta \Delta A_y;
\]

solving the induction equation in terms of the magnetic vector potential \(A\) implies that the condition \(\nabla \cdot \bm{B} = 0\) for the magnetic field \(\bm{B}\) is fulfilled automatically. Here, the usual notation is adopted, where \(\bm{u}\) is the velocity field, \(\rho\) is the plasma density, \(c_s\) is the isothermal sound speed, so that the plasma pressure is \(p = c_s^2 \rho\), \(\Delta = (0, A_y, 0)^T\) is the magnetic vector potential, \(\bm{B} = \nabla \times \Delta\) is the magnetic field, \(\mu_0\) is the magnetic permeability, \(\eta\) is the magnetic diffusivity, \(\mu_0 \sigma\) is the viscous stress tensor with components \(\mu_0 \sigma_{ij} \equiv \mu_0 (\partial u_i / \partial x_j + \partial u_j / \partial x_i)\), \(\nu\) is the kinematic viscosity, and \(t\) is time. Furthermore, \(\bm{J} = \nabla \times \bm{B} / \mu_0\) is the current density and \(\bm{E} = -\nabla \times \bm{B} / \eta\) is the electric field. In the 2-D system, the magnetic vector potential can be gauged in such a way that it has only one component, \(A_y\). Relevant quantities can then be formulated in terms of \(A_y\) as:

\[
B_x = -\partial x A_y, \quad B_z = \partial z A_y, \quad J_y = -\Delta A_y / \mu_0,
\]

\[
\bm{E} \times \bm{B} / \mu_0 = (\bm{u} \cdot \nabla A_y) \nabla A_y / \mu_0 + \eta \Delta A_y \nabla A_y / \mu_0.
\]

2.2 Normalisation

We normalise the equations and quantities with respect to the length unit, \(l_0 = 5\) Mm, the magnetic field strength unit, \(B_0 = 5\) G, and the plasma density unit, \(\rho_0 = 10^{-16}\) g cm\(^{-3}\). \(B_0\) and \(\rho_0\) are typical values in the low quiet-Sun corona, and \(l_0\) is a typical width of a photospheric magnetic source. We take into account that our sources are located above the photosphere, by assuming an expanded source width of 15 Mm (cf. Sect. 2.3.1). The resulting physical units are \(v_0 \equiv B_0 / \sqrt{\pi \rho_0} \approx 1400\) km s\(^{-1}\) for the velocity (this is indeed a characteristic Alfvén speed at our coronal base), \(v_0 \equiv l_0 / v_0 \approx 3.6\) s for time, \(\rho_0 \equiv l_0 v_0 / 7 \times 10^{16}\) cm\(^3\) s\(^{-1}\) for the diffusivities, \(W_0 \equiv B_0^2 l_0 / (4\pi) \approx 2.5 \times 10^{20}\) erg for energy, and \(F_0 \equiv B_0 l_0^2 \approx 1.25 \times 10^{18}\) Mx for the magnetic flux. With this normalisation, the only change to the equations and quantities is to drop \(\mu_0\) in their dimensionless form. We choose our isothermal coronal sound speed as \(c_s = 0.1 v_0 \approx 140\) km s\(^{-1}\). Since \(T = c_s^2 \mu / R\) for an isothermal perfect gas (where \(R \approx 8.3 \times 10^7\) cm\(^3\) s\(^{-2}\) K\(^{-1}\) is the gas constant and \(\mu = 0.6\) the mean specific weight), our isothermal coronal temperature is \(T = 0.01 c_s^2 \mu / R \approx 1.4 \times 10^6\) K.

In the following we will use and plot dimensionless quantities, unless otherwise stated.

2.3 Initial state and boundary conditions

In all our numerical simulations listed in Table 1 we have a uniformly spaced mesh. In the ‘Con’ experiments, we have 199 grid points in \(x\) and 100 grid points in \(z\); in the ‘Dis’ experiments, we have 401 grid points in \(x\) and 100 grid points in \(z\). In all cases, the mesh sizes are \(\delta x = \delta z \approx 0.06\).

Initially, the velocity field is zero in the domain except at the base where we drive our system, and the density is 1 everywhere. During the runs, for the density \(\rho\) we make an extrapolation to the boundaries that corresponds to setting the second normal derivative of \(\rho\) to zero at the boundaries. In order to avoid unphysical behaviour of the density at the base point of the axis (see below), we ensure that the density at the bottom boundary neither drops below nor exceeds the
iments.

In experiments where the inner sources are initially partially connected (Experiments starting with ‘ConEq’), Bottom panel: In the experiments where the inner sources are initially completely disconnected (Experiments starting with ‘DisEq’).

\[
\begin{align*}
\tanh x &= \frac{0.5 \times L_{\text{arc}}}{\alpha_{\text{arc}1}} \\
\tanh \left( -\frac{0.5 \times L_{\text{arc}}}{\alpha_{\text{arc}2}} \right) &- \tanh \frac{0.5 \times L_{\text{arc}}}{\alpha_{\text{arc}3}} + \tanh \frac{0.5 \times L_{\text{arc}}}{\alpha_{\text{arc}4}} \\
A_{y}(2L_{\text{src}}, 0) &
\end{align*}
\]

for \( x \in [0, L_{\text{arc}}], x \in [L_{\text{arc}}, 2L_{\text{arc}}], x \in [2L_{\text{arc}}, L_{x} - 2L_{\text{arc}}], x \in [L_{x} - 2L_{\text{arc}}, L_{x} - L_{\text{arc}}], \) and \( x \in [L_{x} - L_{\text{arc}}, L_{x}], \) respectively.

In the ‘ConEq’ experiments, \( L_{x} = 4L_{\text{arc}} \) and \( \alpha_{\text{arc}1} = \alpha_{\text{arc}2} = \alpha_{\text{arc}3} = \alpha_{\text{arc}4} = 0.353, \) whilst in the ‘DisEq’ experiments, \( L_{x} = 8L_{\text{arc}} \) and \( \alpha_{\text{arc}1} = \alpha_{\text{arc}4} = 0.353, \alpha_{\text{arc}2} = \alpha_{\text{arc}3} = 2.5 \times 0.353 = 0.8825, \) \( 1/\alpha_{\text{arc}i} \) is the maximum magnetic field strength of source \( i. \) Assuming uniformity of the magnetic

2.3.1 Initial magnetic field: connected or disconnected

In all our experiments, we start with an initial magnetic field configuration with four magnetic sources of alternating polarity at the base, each of width \( L_{\text{arc}} = 3. \) We consider a system that is magnetically closed at the top, left and right boundaries, and the magnetic field in the domain above the base is obtained by a potential field extrapolation from the four boundaries (see below). The two outer sources are fixed at all times and the flux connecting them forms an overlying field. On each side of the middle vertical axis (called the ‘axis’), inner and outer sources are connected. The inner sources, which will be pushed together until they fully cancel, are modelled in two main ways, as depicted in Fig. 1 and described below. Table 1 lists all the numerical experiments discussed in this paper, together with the parameters differing in experiments and a brief description of the experiments.

In the ‘ConEq’ experiments, initially all four sources are attached at the base, and equal in both size and flux distribution. In particular, also the inner sources are partially connected such that they marginally touch at the base. In this model, our coronal region is four sources wide and two sources high, i.e. it spans over 60 Mm \( \times \) 30 Mm.

In the ‘DisEq’ experiments, we start off with the inner bipolar sources being completely disconnected from each other in order to model a complete BP. To achieve this initial state, the inner sources have a more flattened flux distribution than the outer ones (the latter being equal to those of the ‘ConEq’ experiments); moreover, they are separated by a gap of the width of four sources so that in this model, our coronal region is eight sources wide and two sources high, i.e. spanning over 120 Mm \( \times \) 30 Mm. However, all sources remain equal in size and total flux magnitude.

More precisely, we model the four sources at the base by setting \( A_{y}(x, 0) \) to
velocity driver at the base, according to Fig. 2. The horizontal velocity driver is imposed as
\[ u_x(x, 0) = u_{\text{driver}} \frac{u_{\text{prof}}(x)}{\max|u_{\text{prof}}(x)|}, \quad x \in [l_{\text{src}}, L_x/2], \]
with the maximum driving speed \( u_{\text{driver}} \), and the driver’s profile \( u_{\text{prof}}(x) \) set to
\[ \left( \frac{L_x}{2} - x \right) \left( \frac{1}{1 + g_{sl} \left( \frac{L_x}{2} - x \right)^2} - \frac{1}{1 + g_{sl} \left( f_{sl} \frac{L_x}{2} \right)^2} \right). \]
The parameter \( g_{sl} \) controls the steepness of the driver’s profile around the axis. We have chosen a rather high value of \( g_{sl} = 7 \) to ensure a reasonable, not too low, driving speed around the axis, where the two inner sources will start to interact. Concerning the parameter \( q \), a lower \( q \) of 0.25 in the ‘DisEq’ experiments (compared to \( q = 2 \) in the ‘ConEq’ experiments) results in a more flattened overall driving profile. For the parameter \( f_{sl} \), we take \( f_{sl} = 0.5 \) in the ‘ConEq’ experiments so that the driver vanishes naturally at \( x = l_{\text{src}} \). In the ‘DisEq’ experiments we take \( f_{sl} = 15/16 \) so that the driver’s speed would still be relatively large at \( x = l_{\text{src}} \), where we then set it to zero. In this way, advection of the whole inner sources is more efficient with the driving profile for the ‘DisEq’ experiments (cf. bottom panel of Fig. 2); in these experiments each of the two inner sources needs to be advected over a distance of 2\( l_{\text{src}} \). The advection time over the length of one source is now about 3/0.005 = 600 at the initial outer edges of the inner sources (if \( u_{\text{driver}} = 0.01 \)) and relatively close to the minimum advection time of 3/0.01 = 300 in large parts over \( x \).

The velocity driver is antisymmetric with respect to the axis,
\[ u_x(x, 0) = -u_x(L_x - x, 0), \quad x \in [L_x/2, L_x - l_{\text{src}}]. \]

We drive the system at three different maximum speeds: \( u_{\text{driver}} = 0.01 \), \( u_{\text{driver}} = 0.05 \), and \( u_{\text{driver}} = 0.1 \), corresponding to about 14 km s\(^{-1}\), 70 km s\(^{-1}\), and 140 km s\(^{-1}\), respectively. Note that these are driving speeds in the lower corona, where the base of our system is located. The driving speed of 0.1 is still only about 0.035 times the initial maximum Alfvén speed, which is \( \approx 2.83 \) (4000 km s\(^{-1}\)) in all models.

Regarding the vertical velocity, in most of our experiments we do not allow for downflows across the base. In some experiments with initially partially connected sources, however, we do not close the base for downflows. In order to let them evolve as freely as possible, the downflows are not imposed in shape or amplitude but the vertical flow is extrapolated from the domain onto the base; no upflows are allowed. If present, the downflow boundary condition is the same as in vRPP05, where it is described in detail.

The magnetic field at the base is determined through \( A_y \) which is advected in time, including also the vertical flow. This means that at the base we solve the induction equation Eq. (3) with zero \( \eta \). Advection of \( A_y \) at the base is done separately from the left and from the right, both up to the axis \( x = L_x/2 \), where the stabilising (but also natural) conditions \( \partial^2 A_y / \partial x^2 \bigg|_{x = 0} = 0 \) and \( \partial A_y / \partial x \bigg|_{x = 0} = 0 \) are set.

At the base of the domain, the magnetic diffusivity \( \eta = 0 \). Inside the domain, \( \eta \) is proportional to the magnitude of the current density, with the proportionality constant equal to 1 so that over time the maximum \( \eta \) varies between \( \approx 0.08 \)

\[ \text{Figure 2. Horizontal velocity driver at the base. Top panel: in the experiments where the inner sources are initially partially connected (Experiments starting with ‘ConEq’). Bottom panel: in the experiments where the inner sources are initially completely disconnected (Experiments starting with ‘DisEq’). In both panels, the driver’s profile is shown for } u_{\text{driver}} = 0.01. \text{ The bottom profile is more suitable for efficient advection (compared to numerical diffusion) of the whole inner sources. This is important in the ‘DisEq’ experiments, where each of the two inner sources has to be advected over a distance equal to the width of two sources.} \]
and $\approx 0.22$ in the different experiments. However, $\eta$ is not allowed to be smaller than 0.005 inside the domain. For the kinematic viscosity we take $\nu = 0.05 - 0.25$.

2.3.3 Unequal flux distribution in the inner sources

We also investigate unequal sources, where the flux distribution is different in the two sources of the inner bipolar. This is achieved by choosing different values for the maximum magnetic field strength of these sources, $1/\alpha_{src1}$ and $1/\alpha_{src2}$. A higher $\alpha$-value means a smaller maximum field strength and, hence, a more flattened flux distribution, but again, all sources are equal in size and total flux magnitude. Due to the asymmetry in the inner bipoles, we explicitly calculate the position ($x_{centre}$) in $x$ where the vertical magnetic field at the base changes sign around the original axis, from the time when the inner sources start to interact onwards. This position is located at the symmetry axis ($x_{centre} = L_x/2$ when the sources are equal). The velocity driver is set between $x = L_{src}$ and $x = 2x_{centre} - L_{src}$, but if the symmetry axis $x_{centre}$ is to the right of the middle vertical axis $L_x/2$, the driver is set to zero at $x = L_x - L_{src}$ so that both outer sources are not moved but fixed in time. In Eqs (5), (6) and (7), as well as in the stabilising conditions on $A_y$ (cf. Sect. 2.3.2), $x_{centre}$ replaces $L_x/2$. Since $x_{centre}$ might change over time, the velocity driver is recalculated (and applied as a boundary condition) at every timestep in the experiments with unequal inner sources.

3 RESULTS

3.1 Vertical magnetic field at the base

We show in Fig. 3 how the vertical magnetic field at the base evolves with time in the ‘Dis’ experiments without downflow. The inner sources at the base (described by $B_z^{t=0}$) are simply advected towards each other until they start to touch between times $t = 600$ and $t = 700$. (When $u_{z}^{t=0} = 0$, $B_z^{t=0}$ is advected via the velocity driver according to $\partial B_z/\partial t = -\partial(u_x B_z)/\partial x$. As discussed in Sect. 2.3.2, the minimum advection time over the length of one source is 300, where the velocity of the driver is maximum. Here and in the following we always rescale the times to the case with $u_{driver} = 0.01$ in order to allow for a better comparison of the experiments with different driving speeds.) Cancellation, however, does not start until a time around $t \approx 800$, as indicated by the percentage values in Fig. 3. Before cancellation becomes efficient around time $t \approx 900$, vertical flux is strongly compressed around the axis where the sources interact and the base becomes field-free.
after which the inner sources become connected by this time each of the two inner sources has been advected. The time evolution of the coronal magnetic field is shown. Pot completely disconnected inner sources are advected towards at the base over a distance of less than one source width so that the inner bipole is still separated at the base by.

Figure 5. Coronal magnetic field evolution in the potential field model. The advection phase lasts until time $t \approx 200$, when an X-point forms at the base. The coronal X-point phase lasts from $t \approx 200$ until $t \approx 1150$, with the maximum X-point height at time $t \approx 400$. At time $t \approx 1150$, the X-point is back on the base. The percentage values are as in Fig. 3. Note the varying range in $x$ in the different panel rows. The outer sources are not shown.

At time $t = 1200$, only about 1% of the vertical flux in the converging inner bipole is left at the base. Figure 3, and in a close-up Fig. 4, illustrates the differences between equal inner sources (black curves), and unequal inner sources of the type described in Sect. 2.3.3 (orange/grey curves). The unequal sources also start to touch at the base between times $t = 600$ and $t = 700$. However, the point ($x_{\text{centre}}^c$) where the converging unequal sources meet at the base moves to the right of the value ($L_x^c/2$) for equal sources. This is because there is almost no flux around the left edge of the right source. Now, the right source is highly peaked around the new axis so that more flux cancels in balance with the left source. This results in quicker cancellation than in the case with equal inner sources at first, but at the end of the coronal reconnection phase (at time $t \approx 1150$), the same amount of flux has cancelled in both cases (about 95%). Thus, unequal sources lead to cancellation starting earlier, but terminating almost at the same time as in the case with equal sources.

3.2 Coronal magnetic field

The time evolution of the coronal magnetic field is shown in Fig. 5 for the potential field run, Experiment DisEqPot. In the first phase, before an X-point is created, the completely disconnected inner sources are advected towards each other. An X-point forms at the base at time $t \approx 200$, after which the inner sources become connected by $B_z$; at this time each of the two inner sources has been advected at the base over a distance of less than one source width so that the inner bipole is still separated at the base by more than two source widths. In the second phase, the X-point rises into the corona until time $t \approx 400$, when the base separation width is less than two source widths, and then moves back towards the base. During this coronal X-point phase there is coronal reconnection: field lines connecting inner and outer sources on each side of the axis reconnect around the X-point, adding flux to the field connecting the inner sources, as well as to the overlying field. This coronal magnetic reconnection, driven by converging sources, causes significant thermal energy release due to Ohmic heating, i.e. a BP (cf. Sect. 3.3). These first and second phases are similar to those of Priest et al. (1994) in the coronal domain.

The third phase, however, which starts when the X-point is back on the base (at time $t \approx 1150$), differs substantially. In our numerical experiments, at this time only about 5% of the flux in the converging inner bipole is left at the base; the cancellation phase starts in the coronal reconnection phase (heating phase, BP phase), during which almost all the base flux of the inner bipole is cancelled. In Priest et al. (1994), cancellation was thought to happen entirely in the third phase, when reconnection occurs at the base. The reason for the difference is that they assumed point sources whereas we resolve them.

In the experiments with initially partially connected sources (cf. vRPP05), as soon as the inner sources are driven towards each other, the already existing coronal X-point starts to drop and reconnection around the X-point sets in. In the coronal reconnection phase, about 90% of the opposite-polarity magnetic flux from the driven inner sources cancels at the base so that again coronal reconnection and base flux cancellation occur mostly simultaneously.

Figure 6 shows that in the MHD field evolution, the creation of an X-point is delayed compared to the potential field evolution (from time $t \approx 200$ to time $t \approx 300$ in the ‘Dis’ experiments without downflows). Hence, the onset of coronal reconnection is also delayed. Furthermore, the maximum X-point height is smaller (assumed at $t \approx 500$ rather than at $t \approx 400$). The delays and lower height are due to the inertia of the plasma and the enhanced pressure gradients associated with a strong density build-up in the inner region where the sources interact. In this region, the magnetic configuration is far from potential. (These effects are reinforced when the driving is done at a higher speed, because the matter build-up in the inner region increases with the driving speed; cf. Sect. 3.4.3). Such behaviour is found in a weak form in the ‘Con’ experiments; in these experiments we start with already partially connected inner sources, rather than with sources that are wide apart so that an X-point exists from the beginning. This limits the density build-up mainly to the region below the X-point. Nevertheless, in both models the differences between MHD field and potential field become smaller as the X-point drops.

In Fig. 6 unequal sources are shown; one can see the stronger, narrower flux concentration in the right source with an (initially) less flattened flux distribution. Consequently, $x_{\text{centre}} > L_x^c/2$ (cf. Fig. 4 and Sect. 3.1), and the axis joining ($x_{\text{centre}}, 0$) and the X-point is inclined to the left.

3.3 Plasma and current densities and flow velocity

The plasma density and flow velocity are shown in Fig. 7 for Experiment DisEqNo0.01. Pushing together the inner bipole means pulling apart each inner source from its neighbouring outer source. This causes an expansion of the flux tube connecting the inner and outer source on each side of the axis. As a result, low-density regions are created in the regions of
Figure 6. Magnetic field lines in Experiment \textit{DisUnNo0.05} with $u_{\text{driver}} = 0.05$ at various times (black lines) and magnetic field lines for potential fields for comparison (orange/grey lines; Experiment \textit{DisUnNoPot}), shown up to times where they differ noticeably. The times are rescaled to the case with $u_{\text{driver}} = 0.01$, i.e. multiplied by 5. Note the repeated zooming-in in the last two panel rows.
Figure 7. Magnetic field lines, plasma density (colours/grey shades) and flow velocity vectors in Experiment \textit{DisEqNo0.01} with $u_{\text{driver}} = 0.01$ at various times. The percentage values are as in Fig. 3.

Figure 8. Same as in Fig. 7 but for Experiment \textit{DisEqNo0.01-open} where the top boundary is open to flow.
Figure 9. Magnetic field lines and current density (colours/grey shades) in Experiment $\text{DisEqNo0.05}$ with $u_{\text{driver}} = 0.05$ at various times. The times are rescaled to the case with $u_{\text{driver}} = 0.01$, i.e. multiplied by 5. The percentage values are as in Fig. 3.

Figure 10. Same as in Fig. 9 but for Experiment $\text{DisEqNo0.05-open}$ where the top boundary is open to flow.
the lower magnetic loops, because the flux tube volume is increasing but the matter in each flux tube is roughly constant. At the same time, matter is compressed in-between the inner sources due to the converging motion of these sources. The resulting large density build-up is enhanced when the driving speed is higher and is reduced when downflows through the bottom boundary are allowed. Accumulated matter is ejected along the axis towards the top boundary, but matter is also accumulated below the X-point. Some of the ejected matter is then moving roughly along field lines of the overlying field and of the upper magnetic loops.

When the top boundary is open to flow (see Fig. 8), the density build-up at the top boundary around the axis is less strong because of a rising outflow (with $u_{\text{z, top, axis}} \leq 0.009$), but it is still visible. A developing diverging horizontal flow along the top boundary (with $|u_{\text{z, top}}| \leq 0.05$) and a forming inflow close to the left and right boundaries (with $u_{\text{z, top}} \geq -0.0165$) distribute matter along the outer overlying field.

Expansion of flux tubes and compression of matter is naturally much less pronounced if we start with a partially connected inner bipole, as is visible in Fig. 11. A direct comparison can be made between Figs 7 and 8 (for Experiment DisEqNo0.01 and DisEqNo0.01-open, respectively), and this figure (for ConEqNo0.01, the corresponding ‘Con’ experiment). In ConEqNo0.01, matter is mainly accumulated below the X-point and plasma ejection along the axis is very weak.

The current density in Experiment DisEqNo0.05 is displayed in Fig. 9. First of all, in the advection phase, currents form around the two (upper) separatrices existing in this phase. Then, in the coronal reconnection phase, the strongest currents form around the X-point, creating a coronal BP via Ohmic heating. Currents build also around and further along the upper separatrices, as well as around the overlying field. The point brightening (the BP) is clearly accompanied by loop brightening.
Figure 12. Magnetic field lines and current density (colours/grey shades) in Experiment ConUnDF0.01 with $u_{\text{driver}} = 0.01$ at various times. The percentage values are as in Fig. 3.

Reversed currents forming above the X-point are associated with a slowing down of the plasma flow emerging from the diffusion region, i.e. the region with high current density around the X-point where magnetic diffusivity is enhanced (Biskamp 1986; see also Priest 1990). Here, the flow is partially diverted to the upper magnetic loops and overlying field. As the X-point reconnection site drops in height, first the flow becomes more aligned with the magnetic loops before becoming slower than the driver. The flow velocity in the domain can be up to roughly twice the maximum driving speed at the base.

When the top boundary is open to flow (see Fig. 10), reverse currents form in regions where the axial flow close to the top boundary is straightening (slightly) the overlying field and switching over to a mainly horizontal flow. The current density around the upper corners has the same sign as in the BP in Fig. 10, whereas it is of opposite sign in Fig. 9. Therefore, in the run with closed top we find a bright outer structure in form of a single loop, whereas in the run with the top open to flow we find a bright outer structure in form of a double loop. We note that of the quantities plotted in Figs 13 and 14 (see the next section), only the kinetic energy is somewhat larger when the top boundary is open to flow; the other quantities (including Ohmic heating) remain basically unchanged compared to the case with closed top.

The current build-up around the X-point is weaker in the ‘Con’ experiments, and is further reduced by a lower driving speed and when downflows are allowed, as is visible in Experiment ConUnDF0.01 in Fig. 12. A downflow transports through the bottom boundary both vertical and horizontal flux, thus reducing magnetic flux in addition to flux cancellation at the base and (numerical) flux diffusion through the base, and hence reducing flux compression, density build-up and current formation around the X-point. Nevertheless, a BP is clearly created; loop brightening occurs also in a weaker form. Note that Fig. 12 is zoomed into the inner region, in order to show the asymmetry due to the unequal inner sources.
3.4 Energetics

3.4.1 Definitions and physical units

In Fig. 13 we plot various quantities as functions of time: the height of the X-point, the magnetic and kinetic energies, the Poynting flux across the base, the reconnection rate, and the Ohmic heating in the whole domain, as well as at the axis. Black solid curves are for Experiment ConEqNo0.01 and red/grey dashed curves are for Experiment DisEqNo0.01. In Fig. 14 the same quantities are plotted for the DisEqNo experiments with different driving speeds, and a comparison of DisEqNo0.01 with the corresponding potential field evolution is made. Definitions and physical units of these quantities are the same as in vRPP05, except for the definitions of the reconnection rate. In the ‘Dis’ experiments, the integration limits in \( x \) and the location of the axis are adjusted. We give here the definitions of the non-normalised quantities and the physical units of the plotted dimensionless quantities where \( \mu_0 \) has dropped out; only the case of equal sources is considered here. For unequal sources, appropriate changes regarding the position of the axis need to be made.

The X-point is located on the axis \(( x = L_x / 2 )\); its height is determined as the height where \( B_x \) changes sign. The X-point height is plotted in units of \( l_0 = 5 \text{ Mm} \).

The magnetic and kinetic energies are

\[
E_{\text{mag}} = \int_0^{L_x} \int_0^{L_z} \frac{B^2}{2 \mu_0} \, dx \, dz ,
\]

and

\[
E_{\text{kin}} = \int_0^{L_x} \int_0^{L_z} \frac{\rho u^2}{2} \, dx \, dz ,
\]

respectively, which we plot in units of \( W_0 / l_0 \approx 5 \times 10^{17} \text{ erg cm}^{-1} \).

The Poynting flux across the base \( z = 0 \) is the rate per unit length at which energy is removed from the system (if negative), or at which energy is injected into the system (if positive). Since we drive the system at the base with zero \( \eta \), the Poynting flux across the bottom boundary is

\[
P_{x=0} = \frac{1}{\mu_0} \int_0^{L_x} (\vec{E} \times \vec{B})_z \, dx \bigg|_{z=0} = \frac{1}{\mu_0} \int_0^{L_x} (-u_x B_z B_z + u_z B_z^2) \, dx \bigg|_{z=0} ;
\]

it is plotted in units of \( W_0 / (l_0 t_0) \approx 1.5 \times 10^{17} \text{ erg cm}^{-1} \text{ s}^{-1} \).

The reconnection rate is the rate at which magnetic flux is reconnected. All the reconnected flux crosses the axis. The reconnected flux crossing the axis above the X-point is the same as the reconnected flux crossing the axis below it. However, some flux below the X-point diffuses through the bottom boundary (due to numerical diffusion) so that in this paper, we define the reconnection rate as the rate at which magnetic flux crossing the axis above the X-point is reconnected, i.e. as

\[
R_{\text{reco}} = \frac{\partial}{\partial t} \int_{h_{\text{Xp}}}^{L_x} A_y \bigg|_{x=L_x/2} \, dz ,
\]

where the height of the X-point, \( h_{\text{Xp}} \), changes with time. \( R_{\text{reco}} \) is plotted in units of \( F_0 / t_0 \approx 3.5 \times 10^{17} \text{ Mx s}^{-1} \). The total reconnected flux is indicated; it is the reconnection rate integrated over the time of the simulated event and in units of \( F_0 = 1.25 \times 10^{18} \text{ Mx} \).

The Ohmic heating in the domain and at the axis are

\[
O_{\text{heat}}^{\text{dom}} = \int_0^{L_x} \int_0^{L_z} \eta \mu_0 J^2 \, dx \, dz ,
\]

and

\[
O_{\text{heat}}^{\text{axis}} = \int_0^{L_x} \eta \mu_0 J^2 \, dz \bigg|_{z=L_z/2} ,
\]

respectively, and we have plotted these energy rates in units of \( W_0 / (l_0 t_0) \approx 1.5 \times 10^{17} \text{ erg cm}^{-1} \text{ s}^{-1} \) and \( W_0 / (l_0^2 t_0) \approx 3 \times 10^{16} \text{ erg cm}^{-2} \text{ s}^{-1} \), respectively. Also indicated is the total heat released in the simulated events, in units of \( W_0 / l_0 \approx 5 \times 10^{17} \text{ erg cm}^{-1} \) (in the whole domain) or \( W_0 / l_0^2 \approx 10^9 \text{ erg cm}^{-2} \) (at the axis).
Poynting’s theorem reflects the energy balance. Using Eqs (10), (8) and (12), and with $\vec{F}_L = \vec{J} \times \vec{B}$ being the Lorentz force, it is

$$P_{z=0} = \frac{\partial}{\partial t} E_{\text{mag}} + O_{\text{heat}} + \int_0^{2l_{\text{arc}}} \int_0^{4l_{\text{arc}}} \vec{u} \cdot \vec{F}_L \, dx \, dz. \quad (14)$$

### 3.4.2 Comparison: initially connected/disconnected bipole

As is visible in Fig. 13, the X-point decreases in height from the beginning in Experiment ConEqNo0.01, while in Experiment DisEqNo0.01 there is no X-point initially. In the ‘Dis’ experiments, the creation of an X-point at time $t \approx 300$ is accompanied by the onset of reconnection, which is very efficient (see the peak in the reconnection rate) and causes a rapid rise of the X-point until a time somewhat before the inner sources start to touch at the base between times $t = 600$ and $t = 700$. The maximum X-point height in DisEqNo0.01 (assumed around $t \approx 500 - 550$) is a little higher than in ConEqNo0.01 (at $t = 0$). During the fall of the X-point, in both cases coronal reconnection has a maximum rate just before cancellation is most efficient. A plasma flow develops but in both experiments the kinetic energy is less than 0.1% of the magnetic energy.

In ConEqNo0.01, the magnetic energy is decreasing during most of the heating phase. (This is true for the experiments starting with partially connected inner sources in general; cf. vRPP05.) This is because during large parts of the heating phase the Poynting flux is negative, which is due to the strong contribution from the flux connecting the inner sources where the driving velocity is high. The magnetic energy is decreasing more quickly than the Poynting flux and when at later times energy is injected into the system by a positive Poynting flux across the base, the magnetic energy is still decreasing for a short while. This is because energy is released in form of Ohmic heating and also because magnetic flux is cancelled. The work done on the field lines is reflected in a positive Lorentz force term.

In DisEqNo0.01 (and the experiments starting with completely disconnected inner sources in general), the Poynting flux is always positive. At the beginning this is due to the absence of any flux connecting the inner sources. At later times, the contribution from the flux connecting the inner and outer sources still dominates due to the different velocity driving profile, but the creation of flux connecting the inner sources causes the Poynting flux to eventually decrease. (The Poynting flux starts decreasing roughly at the same time as when the X-point begins to fall, and when reconnection starts becoming more efficient again. This is somewhat before the inner sources start to touch at the base.) Later, the Poynting flux decreases also because of cancellation.

The positive Poynting flux (from pulling apart the inner and outer sources) causes the magnetic energy to increase. When the Poynting flux becomes smaller, the magnetic energy increases more slowly. Parts of the injected energy goes into Ohmic heating. Again, the work done on the field lines is reflected in a positive Lorentz force term. When cancellation sets in at $t \approx 800$, the magnetic energy starts decreasing. At this time, Ohmic heating is beyond its maximum in the domain, and at its maximum on the axis.

Ohmic heating in the domain occurs from the beginning in both cases, while heating at the axis occurs basically only in the presence of a coronal X-point and coronal reconnection, which in DisEqNo0.01 starts with the appearance of an X-point at time $t \approx 300$. In this experiment, the maximum Ohmic heating is around times $t \approx 700 - 800$ and cancellation starts at time $t \approx 800$. In ConEqNo0.01, the maximum Ohmic heating is around $t \approx 150$ and cancellation starts soon after $t = 0$. Thus, in ConEqNo0.01 the CMF phase begins well before the maximum heating, and in DisEqNo0.01 just after the maximum heating. At the end of the coronal reconnection phase, when the X-point has dropped to the base, some slow reconnection occurs at the base in both cases; Poynting flux, kinetic energy and heating at the axis have basically ceased and magnetic energy is constant. But some Ohmic heating in the domain is still occurring.

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3.4.3 Comparison: different driving speeds and potential

Effects of increasing the driver’s speed in the ‘DisEqNo’ runs, and differences between potential field evolution and MHD field evolution in this model, are illustrated in Fig. 14, as far as quantities regarding reconnection and energetics is concerned. In Fig. 14 all times are rescaled to the case with \( u_{\text{driver}} = 0.01 \) to allow for a better comparison of the runs with different driving speeds. If the whole BP/CMF event were purely dynamical, curves for different driving speeds would lie on top of each other, which is not the case.

A larger driving speed leads to the later creation and rise of an X-point (and thus to a later onset of coronal reconnection), and to a lower maximum X-point height. The same effects are identified when comparing the MHD field evolution with the potential field evolution (cf. Sect. 3.2). They are due to the plasma inertia and a density build-up in the inner region where the sources interact and the X-point is located. Plasma inertia and pressure gradients are non-negligible there due to the forcing-together of the inner bipole, and these forces become stronger with increasing driving speed. Nevertheless, when cancellation becomes efficient the fall of the X-point becomes purely dynamical.

The larger the driving speed is, the larger is the reconnection rate. The reason is that the event modelled is still dominated by the dynamical timescale and the total reconnected flux has to be the same in all experiments with the same initial state (because the final states are then the same as well). Naturally, the higher the driving speed is, the higher are the kinetic energy and Poynting flux. A larger driving speed leads also to larger Ohmic heating; both heating rates and total heat output are increased with the driving speed (see Sect. 3.4.4). The current build-up is stronger because the currents cannot be dissipated quickly enough compared to the larger driving speed. Of course, the BP lifetime and cancellation time decrease with increasing driving speed (see Sect. 3.4.5), because the BP and CMF events happen roughly on a dynamical timescale.

The time evolution of the magnetic energy in the MHD runs behaves similarly to the potential run, implying little free magnetic energy is created and magnetic energy conversion to heat is very efficient. Also, the magnetic energy evolution is almost completely dynamical, i.e. almost independent of the driving speed in the rescaled picture of Fig. 14. After all, the initial and final magnetic field configurations are the same in all these runs.

3.4.4 Total heat outputs and maximum heating rates

Visible in Fig. 13, the total heat released in our domain is considerably higher in Experiment ‘DisEqNo’ than in Experiment ‘ConEqNo’. This is because ‘DisEqNo’ covers a complete BP event and ‘ConEqNo’ does not. In order to estimate the total heat produced in a 3-D domain, we assume uniformity of the Ohmic heating in the y-direction, so we multiply the total heat output in our 2-D domain (0.15 and 0.56, in units of \( 5 \times 10^{17} \text{ erg cm}^{-1} \)) by the width of two sources. The resulting total released heat is about \( 8.4 \times 10^{26} \text{ erg} \) in ‘DisEqNo’, while it is only about \( 2.25 \times 10^{26} \text{ erg} \) in ‘ConEqNo’. Thus, the released energy in ‘DisEqNo’ compares quite well with the energies of coronal XBPs deduced from observations (see Sect. 1).

Comparing the heating rates in ‘DisEqNo’ with observed ones, we multiply the maximum heating rate in our 2-D domain (0.00115 in units of \( 1.5 \times 10^{17} \text{ erg cm}^{-1} \text{ s}^{-1} \)) by the width of two sources. This gives a maximum heating rate in the corresponding 3-D domain of \( \approx 5 \times 10^{23} \text{ erg s}^{-1} \) (compared to \( \approx 2.7 \times 10^{23} \text{ erg s}^{-1} \) in ‘ConEqNo’; cf. vRPP05). This lies within the observed range of heating rates from BPs of about \( (3 - 10) \times 10^{23} \text{ erg s}^{-1} \) (Habbal & Withbroe 1981). When reduced to a 1-D vertical slab (i.e. averaged in x-direction and so: divided by the width of four sources), our maximum heating rate provides \( \approx 3 \times 10^{4} \text{ erg cm}^{-2} \text{ s}^{-1} \) heating, which is 10% of what is required to maintain the temperature of the quiet-Sun corona according to Withbroe & Noyes (1977), and almost twice as much of what is found in ‘ConEqNo’ (cf. vRPP05). This estimate assumes that a single BP of the type of Experiment ‘DisEqNo’ occurs in the domain considered, at a given time.

When \( u_{\text{driver}} = 0.05 \) (Experiment ‘DisEqNo’; see Fig. 14), the total heat output in our 2-D domain is 0.669 in units of \( 5 \times 10^{17} \text{ erg cm}^{-1} \), so that the total released heat is \( \approx 10^{27} \text{ erg} \) in the 3-D domain of the y-length of \( 2 \text{l}_{\text{src}} \), compatible with observations. The maximum heating rate in our 2-D domain is 0.008 in units of \( 1.5 \times 10^{17} \text{ erg cm}^{-1} \text{ s}^{-1} \). In the 3-D domain this leads to \( \approx 3.6 \times 10^{24} \text{ erg s}^{-1} \), which is a few times too high, and in the 1-D vertical slab to \( \approx 2 \times 10^{15} \text{ erg cm}^{-2} \text{ s}^{-1} \), which is also too high, namely about 65% of the energy loss rate in the quiet-Sun corona instead of the observed \( \geq 15\% \) due to XBPs caused by CMFs (see Sect. 1). For \( u_{\text{driver}} = 0.1 \) (Experiment ‘DisEqNo’), we find \( \approx 1.2 \times 10^{27} \text{ erg} \) for the total released heat and \( \approx 9 \times 10^{24} \text{ erg s}^{-1} \) for the maximum heating rate, both in the 3-D domain, and \( \approx 5 \times 10^{12} \text{ erg cm}^{-2} \text{ s}^{-1} \) for the maximum heating rate in the 1-D vertical slab. Thus, even for this high driving speed the total heat output is only slightly above \( 10^{27} \text{ erg} \), but the maximum heating rates are far higher than observed.

The scaling of the maximum Ohmic heating with \( u_{\text{driver}} \) is with a power of \( \approx 1.2 \) in the ‘ConEqNo’ runs (both in the domain and at the axis; cf. vRPP05), and in the ‘DisEqNo’ runs of Fig. 14 with a power ranging between 1.2 and 1.5.

3.4.5 BP lifetimes and cancellation times

In Experiment ‘ConEqNo’, the coronal X-point phase (BP phase) goes from \( t = 0 \) until \( t \approx 400 \), i.e. the BP lifetime is about 25 minutes. The CMF phase goes from \( t \approx 50 \) until \( t \approx 650 \), when 99% of the vertical base flux in the inner bipole is cancelled, i.e. the cancellation time is about 35 minutes. Hereby, about 90% of the flux is cancelled by the end of the BP phase. Both BP lifetime and cancellation time decrease roughly dynamically with increasing driving speed.

In Experiment ‘DisEqNo’, the coronal X-point phase (BP phase) goes from \( t \approx 300 \) until \( t \approx 1150 \), i.e. the BP lifetime is about 50 minutes (twice as long as in ‘ConEqNo’). The CMF phase goes from \( t \approx 800 \) until \( t \approx 1200 \), when again 99% of the vertical base flux in the inner bipole is cancelled, i.e. the cancellation time is about 25 minutes (shorter than in ‘ConEqNo’). Thus, cancellation starts here well into the BP phase (and also well after the maximum X-point height at \( t \approx 500 \) – 600), but as much as about 95% of the flux is cancelled by the end of the BP phase. Again, BP lifetime and cancellation time decrease roughly dynamically with increasing driving speed.
4 CONCLUSIONS

We have modelled a complete CMF and performed various 2-D numerical MHD simulations, including experiments covering an associated complete coronal XBP (denoted ‘Dis’) and experiments missing out the first part of a BP (denoted ‘Con’). Our simulation results show that the heating and cancellation phases greatly overlap in time, with cancellation starting during the heating phase and terminating after coronal magnetic reconnection has ceased. At the end of the BP phase, about 90 to 95% of the flux in the converging magnetic bipole (the CMF) has cancelled at the base, where the higher flux cancellation during the BP phase is in the ‘Dis’ experiments.

Point brightening via Ohmic heating around the coronal X-point reconnection site is accompanied by loop brightening. Matter and flux compression, i.e. density and current build-up, and therefore heating, are stronger in the ‘Dis’ experiments than in the ‘Con’ experiments, and also stronger in the absence of downflows. Ohmic heating is also larger for a larger driving speed because the dissipation of the formed currents cannot keep up with the higher driving speed.

Comparing the MHD field evolution with the potential field evolution, we find that the plasma inertia and enhanced pressure gradients in the interaction region delay the onset of coronal reconnection and lower the maximum X-point height in the ‘Dis’ experiments. These effects are stronger when the driving is at a higher speed.

Coronal reconnection has a maximum rate just before cancellation is most efficient. Naturally, a larger driving speed leads to a faster reconnection rate.

The magnetic energy dominates greatly over the kinetic energy. In the ‘Con’ experiments the Poynting flux is negative, and the magnetic energy decreasing, during large parts of the heating phase. In the ‘Dis’ experiments the Poynting flux is always positive and the magnetic energy only decreasing when cancellation sets in. Little free magnetic energy is created in all models.

In the ‘Dis’ experiments without downflows, the total heat output is high enough for a coronal XBP, namely around $10^{27}$ erg. The maximum heating rates in the domain and at the axis scale as $v_{\text{driver}}^{1,2,\cdots,1,2};$ they are comparable with observed heating rates fromXBPs caused by CMFs when $v_{\text{driver}} \approx 0.015$. BP lifetimes and cancellation times are on a dynamical timescale.

Future work will comprise the improvement of the thermodynamics by including appropriate heating and cooling terms in the energy equation, and the inclusion of the stratified lower atmosphere linking the corona to the photosphere, before extending the simulations to 3-D.

ACKNOWLEDGMENTS

Use of the supercomputer SGI 3800 in Linköping and of the PPARC supported supercomputer in St Andrews is acknowledged, together with financial support from PPARC on the St Andrews Solar Rolling Grant.

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