Multiply-Connected Source and Null Pairs

C.E. PARNELL[1]
[1] *School of Mathematics and Statistics, University of St Andrews, North Haugh, St Andrews, Fife, KY16 9SS, U.K.* (clare@mcs.st-and.ac.uk)

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Abstract. The magnetic fields within the solar atmosphere have a complex topology due to the fragmentary nature with which they thread the solar surface. The topologies of the potential magnetic fields containing only a few (up to four) point photospheric sources have been classified. For small numbers of sources determining the connectivity of source pairs is equivalent to counting the number of flux domains. As the numbers of sources increase this, however, is no longer the case. Instead, a pair of connected sources can have more than one distinct flux domain linking them. We call these *multiply-connected source pairs*. Pairs of nulls connected by more than one separator are called *multiply-connected null pairs*. Multiply-connected source and null pairs go hand-in-hand such that two separators connecting the same pair of nulls immediately implies multiple flux domains linking the same source pair and vice versa.

It is found that multiply-connected source pairs are common in not only fairly complex potential magnetic fields, but more interestingly in the resistive-MHD evolution of magnetic fields both simple and complex. Magnetic energy release is often significant around separators. Thus fields with multiply-connected source pairs which naturally have more separators have both (i) more sites for intense energy release and (ii) are likely to release energy more quickly than other magnetic fields. Moreover, the combination of multiply-connected source and null pairs can give rise to a situation where flux is reconnected repeatedly between two flux domains.

Keywords: Sun: magnetic fields; solar corona

1. Introduction

The Sun's surface is threaded by a continuum of magnetic flux that is directed both into and out of the Sun. These regions of magnetic flux appear to group themselves according to their polarity into magnetic fragments. The fragments have a vast array of sizes, ranging from sunspots, which can have a few times 10^{20} Mx of flux (Schrijver and Harvey, 1994), down to intra-network fragments, which have as little as 10^{16} Mx of flux (Wang et al., 1995) and smaller. Indeed magnetogram images of the quiet-Sun photosphere reveal, with increasing resolution, more and more flux sources. That is, in between the obvious flux concentrations the magnetic field does not appear to simply fall off in a Gaussian manner modified by white noise, but is instead full of smaller magnetic fragments of both polarities. In turn these small fragments are again surrounded by a sea of 'noise' which may well turn out to host a mixture of even smaller fragments of both polarities.

Magnetograms show that these fields are very dynamic in nature (Schrijver and Zwaan, 2000) so, not surprisingly, the topology of the magnetic field that fills the solar atmosphere is highly complex. Understanding the topology of the Sun's magnetic field, as well as its evolution, is central to explaining the behaviour of the solar atmosphere. This is because magnetic reconnection involves the transfer of flux between flux domains. So knowing how many flux domains there are, where their boundaries (known as separatrix surfaces) are, indicates where and how many possible sites of reconnection there are. In particular, changes of topology (or bifurcations) are important as they signify the creation of a new site of reconnection or the disappearance of an old site. For instance, Close et al. (2004; 2005a) have considered the evolution of the magnetic topology of the quiet-Sun's coronal magnetic fields and have shown that there is significantly more reconnection in the corona than one would anticipate from emergence and cancellation of magnetic flux in the photosphere alone. Indeed, they find that typically all the magnetic connections in the quiet-Sun are changed every 1.4 hrs - i.e., the quiet-Sun coronal recycling time just a tenth of the photospheric recycling time. However, further work in this area is important for understanding the dynamic nature of the solar corona.

The magnetic fields on the Sun's surface may be modelled as discrete point or finite sources or as a continuous field. Given the highly mixed nature of the magnetic field in the weak flux regions it is debatable whether finite sources or a continuous field gives the best representation of the Sun's magnetic field.

Modelling the photospheric field as discrete finite sources gives a better representation of the Sun's magnetic field than point sources, however, point sources are far easier to implement and so they are more commonly used. In both cases the topology of the magnetic field is defined by its null points, most of which occur on the base, the separatrix surfaces and separators that extend from these nulls. Collectively these features are known as the magnetic skeleton (Bungey, Titov, and Priest, 1996; Priest, Bungey, and Titov, 1997). A separatrix surface consists of field lines that extend from a source to a null point. The null is said to be positive if the field lines are directed out from the null and negative if they are directed into it (e.g., Parnell et al., 1996). Separatrix surfaces bound flux domains. Separators are the intersection of two separatrix surfaces from opposite polarity nulls and are field lines that border four flux domains. In three-dimensions they are important sites for magnetic reconnection (e.g., Longcope, 1998; Galsgaard, Parnell, and Blaizot, 2000; Longcope, 2001; Parnell and Galsgaard, 2004; Galsgaard and Parnell, 2005; Longcope, 2005; Haynes et al., 2007).

Where the field through the surface is modelled by a continuum of sources the topology of the magnetic field will often be less complex due to the loss of the majority of surface nulls. Here, instead, by considering the geometry of the field, a quasi-skeleton may be found including quasi-separatrix surfaces and quasi-separators or hyperbolic flux tubes (Titov, Priest, and Démoulin, 1993; Priest and Démoulin, 1995; Titov, Hornig, and Démoulin, 2002). It is clear that heating and reconnection can also occur at quasi-separatrix surfaces and hyperbolic flux tubes where the gradient of the mapping of field lines from one part of the photosphere to another is large whereas for separatrix surfaces it is infinite (Titov, Galsgaard, and Neukirch, 2003; Galsgaard, Titov, and Neukirch, 2003; Hornig and Priest, 2003; Priest, Hornig, and Pontin, 2003; De Moortel and Galsgaard, 2006a;b). In these types of magnetic fields sources and flux domains may again be identified with flux domains bounded by quasi-separatrix surfaces. It is therefore not unreasonable to assume that multiply-connected source pairs and multiply-connected quasi-separators also exist in fields with quasi-skeletons. Throughout this paper we will consider magnetic fields from either point or discrete sources, but the results are, in general, applicable to magnetic fields due to continuous sources as well.

The topologies of simple potential magnetic fields associated with three and four point sources on a plane, and the bifurcations through which these topologies are linked, have been identified by Brown and Priest (1999a; 1999b), Beveridge, Priest, and Brown (2002), Beveridge, Longcope, and Priest (2003), Beveridge, Priest, and Brown (2004), Pontin and Priest (2003) and Maclean et al. (2005). In particular, they discuss *global bifurcations*, in which the numbers of nulls remain fixed and *local bifurcations*, in which pairs of nulls are created or destroyed during a change of topology. Longcope (2005) has written a comprehensive review of these topologies. In such simple topologies the number of connected source pairs equals the number of flux domains.

More complex potential magnetic fields, extrapolated from magnetograms, have been considered by Close, Parnell, and Priest (2005b) and Barnes, Longcope, and Leka (2005). In each of these, multiply-connected source pairs and multiply-connected null pairs were discovered. In each paper only one magnetogram was considered, but ten multiply-connected sources were found in one of them. This suggests that multiply-connected source pairs are likely to be common in magnetic topologies containing lots of magnetic sources. An alternative form of complexity can be introduced by considering a global magnetic field resulting from sources on a spherical, as opposed to planar, surface. Here, just four sources can give rise to a multiply-connected source pair (Maclean et al., 2006). Furthermore, recent numerical results from a resistive MHD experiment by Haynes et al. (2007) have shown that multiply-connected source pairs commonly arise during the MHD evolution of simple magnetic fields involving just four magnetic sources.

Little, however, is known about multiply-connected source pairs and their flux domains. Indeed, Barnes, Longcope, and Leka (2005) even refer to them as ‘redundant domains’ suggesting in some way that they are less important than other flux domains. Thus, in this paper we focus on multiply-connected source pairs and their flux domains and consider their importance. First, in Section 2, we review flux domains and define multiply-connected source and null pairs. In Section 3, potential multiply-connected source pairs are considered with examples given for the main types of doubly-connected source pairs. Section 4 looks at the multiply-connected source pairs that arise through the MHD evolution of a magnetic field. Section 5 discusses the importance of these types of source and null pairs and considers whether it is possible to predict what type of flux domains you have from knowing information about the number and type of elements in the skeleton. Finally, the conclusions are given in Section 6.

2. Flux Domains and Multiply-connected Source Pairs

As already said, the key topological elements of any complex 3D magnetic field are encompassed in its magnetic skeleton. The skeleton is useful as it identifies the boundaries of flux domains (separatrix surfaces) and the border between four flux domains (separators) which are key sites of magnetic energy release due to reconnection. We must, however, define what we mean by a *flux domain*. A magnetic *flux domain* (Longcope and Klapper, 2002; Beveridge and Longcope, 2005) is a simply-connected volume of flux which is bounded by either separatrix surfaces or the source plane and contains field lines that connect the same two sources. By *simply-connected* we mean that the field lines are topologically equivalent and can be continuously deformed from one to another without leaving the volume.

There are four types of generic flux domains. *Isolated domains* are bounded by a single, unbroken separatrix surface whose field lines connect to a single source. *Ordinary or photospheric domains* are bounded by several separatrix surfaces which intersect in a series of separators. Both of these two types of domain have a footprint on the source plane. *Coronal domains* are engirdled by a ring of separators which lie entirely above the source plane and so have no footprint on the source plane. Finally, *semi-coronal domains* also involve a ring of separators which lies above the source plane save for a pair of separators from an upright null that lie in the source plane and link to two prone nulls. See, for example, Lau and Finn (1990), Longcope (2001), Longcope and Klapper (2002) or Close, Parnell, and Priest (2005b) for further details on flux domains.

Naturally, to find such domains one first traces field lines and determines which opposite-polarity source pairs are connected. Clearly, one source can connect to one or many other sources (see Schrijver and Title, 2002 and Close et al., 2003 for a statistical analysis on the numbers of other sources), but it is not so obvious that one connected source pair may be joined by several discrete flux domains (Longcope and Klapper, 2002).

If all the field lines linking one source pair form a flux domain (that is they are topologically equivalent and can be continuously deformed from one to another without breaking) then the source pair is said to be *simply-connected*. Such domains can be of any of the above four generic types. However, if the field lines connecting a source pair are not topologically equivalent, then there must be more than one flux domain linking them and the source pair is known as *multiply-connected*. Multiply-connected source pairs are inextricably linked to separators, as explained below, and so isolated domains cannot link these source pairs.

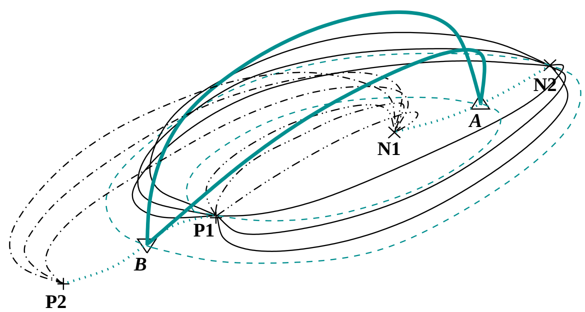

Figure 1. Three-dimensional sketch of part of a magnetic configuration containing a multiply-connected source pair. The positive sources (P1 and P2), negative sources (N1 and N2) and positive/negative nulls (A and B) are indicated by a $+$, $\times$ or a $\triangle/\bigtriangledown$, respectively. The dashed lines and dotted lines show the separatrix and spine fieldlines lying in the source plane, respectively. The two thick solid lines represent the separators that link the two nulls. Example field lines are drawn in the two flux domains linking P1-N2, the multiply-connected source pair (thin solid), in the coronal domain connecting P2-N1 (dot-dashed) and in the ordinary domain P1-N1 (dot-dot-dashed). The field lines linking P2-N2 are not shown, but they arch over all the other domains.

Figure 1 shows a sketch of a magnetic configuration containing both simply and multiply-connected source pairs. In Figure 1 two ordinary flux domains link P1-N2, the multiply-connected source pair (solid curves). These flux domains are separated by a coronal flux domain linking P2-N1 (dot-dashed curves), which is bounded by two separators linking the null pair B-A. Also shown are field lines in the ordinary domain P1-N1 (dot-dot-dashed curves).

Flux domains are defined by their location relative to all separatrix surfaces in the system, i.e., for one flux domain to contain the same flux as another it must lie inside (and outside) the same separatrix surfaces as the first domain. Consider, say, a separatrix surface from a positive null B which separates flux P1 from flux P2. A second separatrix surface from a negative null divides flux N1 from flux N2. The intersection of these two separatrix surfaces creates one separator from B to A and divides the flux into four

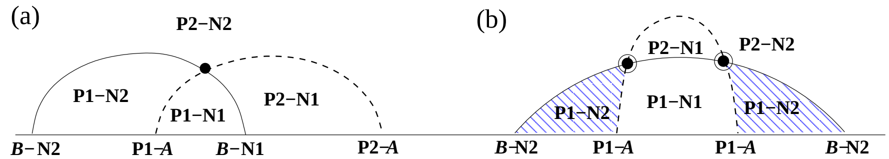

Figure 2. Sketches of a cut through two separatrix surfaces, one extending from a positive null (solid) and the other from a negative null (dashed). (a) The separatrix surfaces intersect only once to create a single separator (dot) and four flux domains each connecting different source pairs. (b) The separatrix surfaces intersect twice creating a multiply-connected source pair P1-N2 (shaded domains) and two separators linking the same null pair (circled dots) creating a doubly-connected null pair. A coronal domain, P2-N1, always exists with every multiply-connected null pair.

domains each with flux connecting different source pairs. Figure 2a shows a two-dimensional cut through the separatrix surfaces of such a setup. The intersection of the separatrix surfaces from B and A with the plane are shown as solid and dashed lines, respectively. Clearly, if these separatrix surfaces only intersect each other at one separator then there is only one P1-N2 flux domain no matter how many other separatrix surfaces there are in the system. The only way of creating another P1-N2 flux domain is for the two separatrix surfaces to intersect again which obviously results in a second B-A separator (Figure 2b). The three-dimensional configuration for this field is shown in Figure 1 with the cut, shown in Figure 2b made along a line approximately perpendicular to the axis of the sources and lying between sources P1 and N2.

The same pair of nulls connected by m separators is known as a *multiply-connected null pair* of multiplicity m. As shown above multiply-connected source pairs are always associated with multiply-connected null pairs. Furthermore a double separator implies that there must be at least one domain that is wholly surrounded by separatrix surfaces and ringed by separators, i.e. where there are multiply-connected source and null pairs there is always at least one coronal domain. In Figure 2 and Figure 1 the coronal domain is P2-N1.

Beveridge and Longcope (2005) propose an equation that provides a check to see if a consistent set of elements has been found for the magnetic skeleton of a particular magnetic topology, namely,

$$\mathcal{D} = X + S - N_c - 1, \tag{1}$$

which relates the number of flux domains ($\mathcal{D}$), separators (X), sources (S) and coronal nulls (N_c). Here, however, we rewrite this equation in order to count multiply-connected source pairs more overtly. To do this we expand the total number of domains ($\mathcal{D}$) in terms of a sum of the numbers of flux domains between all connected source pairs of multiplicity n ($\mathcal{D} = \sum_n nD_n$)

such that the Beveridge-Longcope equation becomes

$$\mathcal{D} = \sum_n nD_n = X + S - N_c - 1 \; . \tag{2}$$

This equation holds for magnetic topologies that are balanced and have discrete, resolved sources.

In particular, we note that the number of connected source pairs is equal to $\sum_n D_n$, regardless of the multiplicity of the source pairs. Thus, if the right-hand side of Equation 2 is equal to the number of connected source pairs then clearly all connected source pairs must be simply connected. If, however, the right-hand side is greater, then (assuming the number of connected source pairs has been determined correctly) there must be multiply-connected source pairs. A possible check for this may be made by checking the multiplicity of the null pairs (although determining the number of separators is not necessarily simple) since wherever there is a multiply-connected source pair there will also be a multiply-connected null pair. Thus, the Beveridge-Longcope equation is useful in identifying fields with multiply-connected domains.

3. Multiply-Connected Source Pairs in Potential Fields

Doubly-connected pairs, or source pairs with multiplicity two, have been observed in a number of magnetic topologies (Longcope and Silva, 1998; Close, Parnell, and Priest, 2005b; Barnes, Longcope, and Leka, 2005; Maclean et al., 2006). Close, Parnell, and Priest (2005b) looked in detail at the structure of the coronal field produced by a complex set of flux sources observed in a photospheric magnetogram and found that one source pair was connected by two ordinary domains and another source pair by two coronal domains. They explained why the two ordinary domains arose, but did not discuss the configuration containing the coronal source pair. A third type, the ordinary-coronal doubly-connected source pair, can also been found in a potential configuration as we demonstrate below. Thus, there are at least three flavours of a doubly-connected source pair, namely ordinary-ordinary (OO), coronal-coronal (CC) and ordinary-coronal (OC) where the name identifies the type of the two flux domains connecting the source pair. Below, in Sections 3.1-3.3, we describe examples of potential magnetic topologies that contain these flavours of doubly-connected source pair. Particular examples of each of the potential fields are given in Section 3.4. Finally, Section 3.5 demonstrates that fields involving multiply-connected source pairs are generic and shows how they can arise.

Note, that in practice any finite region of the corona will consist of unbalanced flux, since in general there will be some flux leaving or entering the region. On the other hand the global corona has to be in flux balance. Thus when modelling a finite region, such as considered below, it is natural to suppose the flux is unbalanced and to balance it by an equal and opposite flux at large distances. For simplicity we place this balancing flux at infinity which has the advantage that the resulting topologies do not depend on the distance of this source or its location. This is a good approximation provided such flux is at a distance much larger than the dimensions of the region under consideration. A further advantage is that such a region is often bounded by a finite separatrix surface that separates the flux that closes locally from that which extends to infinity. Thus the topologies discussed in Sections 3.1-3.3 all have an unbroken separatrix surface bounding the flux of interest. Section 3.5, however, discusses topologies involving multiply-connected source pairs that are not bounded by an unbroken separatrix surface, as well as one which is in flux balance.

3.1. OO Multiply-Connected Source Pair

An *OO source pair of multiplicity two* consists of two *ordinary* domains connecting the same source pair. Figure 3 shows sketches of a footprint and cross-section of a potential field containing such a source pair. In this example, there are two positive and five negative sources (the fifth the balancing source at infinity). No coronal or upright nulls are present, but there are five prone nulls: one positive and four negative (Figure 3a). There are only separators between three of the four null pairs, but there are four separators, so one of these null pairs must be multiply connected (Figure 3c). Indeed, there are two separators connecting $B1$-$A2$. Hence, from the Beveridge-Longcope Equation the total number of domains predicted is

$$\mathcal{D} = X + S - N_c - 1 = 4 + 7 - 0 - 1 = 10\ .$$

There are a maximum of ten source pairs, but only nine are connected (P2 is unconnected to N_∞) and so one source pair must be multiply-connected.

$$\mathcal{D} = \sum_n nD_n = 10, \quad \text{and} \quad \sum_n D_n = 9,$$

$$\text{therefore} \quad D_1 = 8 \quad \text{and} \quad D_2 = 1.$$

Figure 3a shows a sketch of the footprint, with the separators included, for this potential field. Here, it is not clear that any source pair is multiply-connected, but with just eight source-pair connections visible, clearly, there must be at least one coronal domain. Figure 3d shows a vertical cut taken along the horizontal solid line in Figure 3a and clearly shows that the source pair P1-N3 is linked by a coronal domain. This domain divides the flux linking the P2-N2 source pair (shaded domains) making it multiply connected

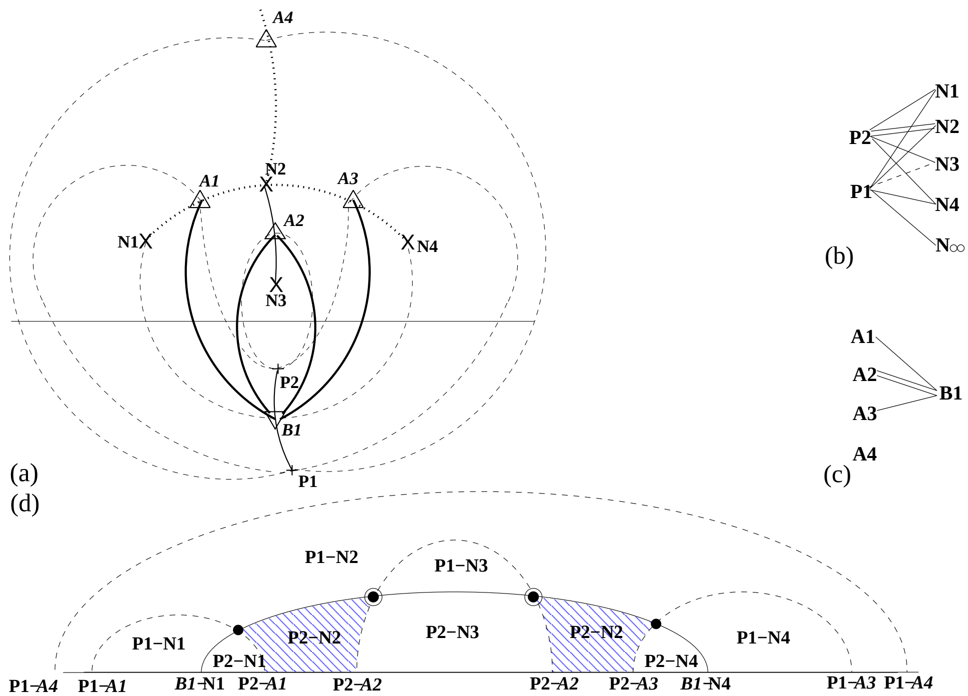

Figure 3. (a) A sketch of the footprint for a potential magnetic field with seven sources and an OO multiply-connected source pair showing positive (+) and negative (×) sources, negative (△) and positive (▽) nulls, separatrix surfaces (dashed), spines (dotted) and separators (thick solid). (b) Domain and (c) null graphs for this magnetic configuration: the former shows ordinary (solid) and coronal (dashed) domains whilst the latter shows separators (solid). (d) A sketch of a vertical cross-section along the solid line through (a) showing the two flux domains linking the multiply-connected source pair (shaded). The separatrix surfaces from positive (B) nulls (solid) and negative (A) nulls (dashed) are shown. Separators are indicated by solid dots. Circled dots indicate that the separator is part of a multiply-connected null pair. The domains are labelled with their source pair and the separatrices are labelled with their source-null pair.

(Figure 3b). This type of configuration was first described by Close, Parnell, and Priest (2005b). It has also been found by Maclean et al. (2006) who called it a *dual-intersecting state*. This type of multiply-connected domain readily arises (see Section 3.5) and is likely to be the most common type of doubly-connected source pair in potential magnetic fields.

3.2. CC Multiply-Connected Source Pair

In a *coronal-coronal source pair of multiplicity two* the domains of the multiply-connected source pair are both purely coronal. In the example shown in Figure 4 there are in fact two multiply-connected sources pairs. One source pair has two ordinary domains the other has two coronal domains.

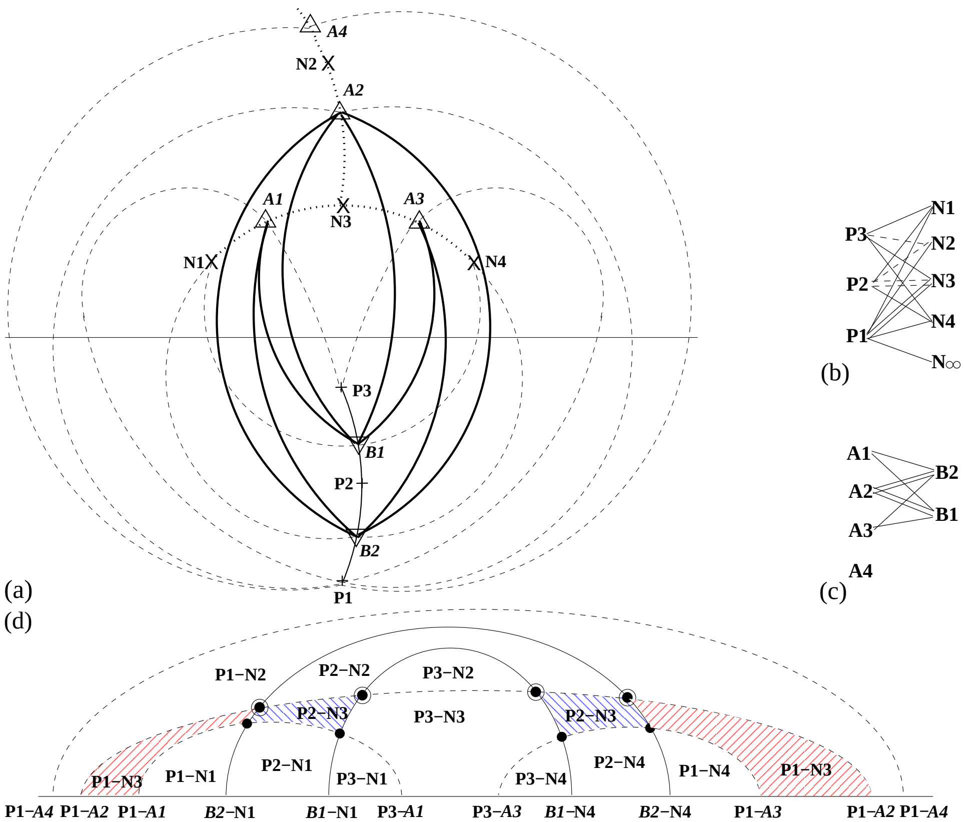

Figure 4. Sketches of (a) the footprint, (b) domain and (c) null graphs and (d) a vertical cross-section of a potential magnetic field that has eight sources and a multiply-connected source pair linked by two coronal domains (downwards cross-hatch) and a multiply-connected source pair linked by two ordinary domains (upwards cross-hatch). Notation is the same as those in Figure 3.

We are interested in the latter which does not always have to occur in conjunction with the former.

In this example there are three positive and five negative sources (one at infinity) with the flux in P1>P2>P3. The negative fluxes are ordered such that N3>N2>N1 and N4. Again, the only nulls formed are prone with four negative and two positive found (Figure 4a). There are six connected null pairs (the distant negative null $A4$ is unconnected) linked by a total of eight separators indicating that there must be at least one multiply-connected null pair (Figure 4c). Equation 2 determines the total number of domains to be,

$$\mathcal{D} = X + S - N_c - 1 = 8 + 8 - 0 - 1 = 15\ .$$

However, there are only 13 connected source pairs.

$$\mathcal{D} = \sum_n nD_n = 15, \quad \text{and} \quad \sum_n D_n = 13.$$

Clearly, here there are two options $D_1 = 12$ and $D_3 = 1$ or $D_1 = 11$ and $D_2 = 2$. Thus, there must be at least one multiply-connected source pair. We discuss later (Section 5.2) if it is possible to say by just knowing the separators which of these is correct.

Connections for 10 of the source pairs can be seen in the footprint for this field (Figure 4a). A sketch of the vertical cut (Figure 4b) along the horizontal line in Figure 4a reveals the missing source-pair connections: coronal domains P2-N2, P2-N3 (twice) and P3-N2. Furthermore the pair P1-N3 is multiply connected with two ordinary domains due to the fact that P2 and P3 are both connected to N2, as well as N3. The domain and null graphs for this configuration are given in Figure 4b and Figure 4c, respectively.

A similar configuration involving a CC multiply-connected source pair was found by Close, Parnell, and Priest (2005b) in a potential extrapolation of a quite-Sun magnetogram, although the architecture of the magnetic fields in their case must have been different as it had no companion OO multiply-connected source pair.

3.3. OC Multiply-Connected Source Pair

The final type of doubly-connected source pair is the *OC multiply-connected source pair*, which has both an *ordinary* and a *coronal* domain. As shown in Figure 5, such a magnetic topology is easily realised with a similar configuration to that for the CC source pair by simply moving and decreasing the flux in N4 (see Section 3.4). Hence, there are three positive and five negative sources creating four positive and two negative nulls (Figure 5a). The number of separators, though, is now just six and only one is a doubly-connected null pair (Figure 5c). A total of twelve connected source pairs are found although $\mathcal{D}$ is predicted, by Equation 2, to be thirteen, hence there must be one multiply-connected domain of multiplicity two (Figure 5b). The cross-section (Figure 5d) clearly shows these thirteen domains. Eleven of these domains are ordinary, but two are coronal: P2-N3 and P3-N2 (Figure 5b). P2-N3 is also linked by an ordinary domain so is multiply-connected.

3.4. Examples of Multiply-Connected Fields

In the above sections, we simply provide idealised sketches of what the magnetic topologies would look like in each case. Here, we give examples of actual potential magnetic fields that contain these multiply-connected source pairs. The topologies of the actual magnetic fields are the same as

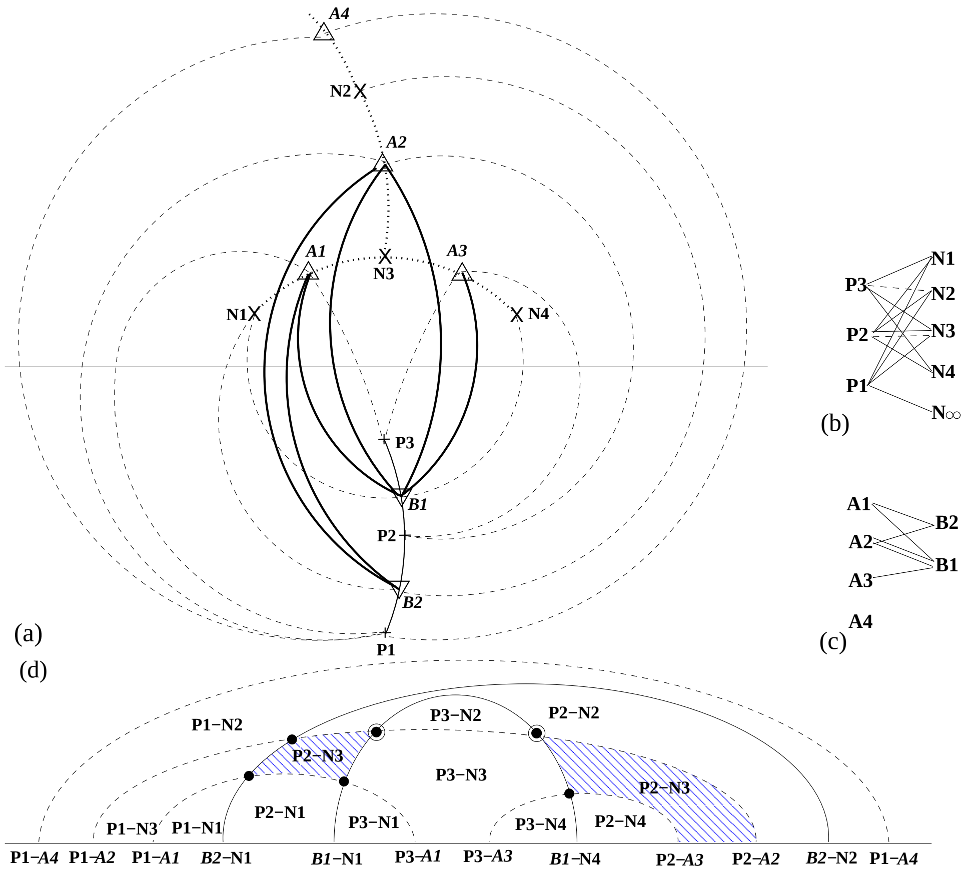

Figure 5. Sketches of (a) the footprint, (b) domain and (c) null graphs and (d) a vertical cross-section through a potential magnetic field that has eight sources and a multiply-connected source pair linked by an ordinary and a coronal domain (shaded). Notation is the same as those in Figure 3.

those described above, but cross-sections through them reveal flux domains of different shapes and sizes than illustrated in the sketches.

An example of a potential field containing an OO multiply-connected source pair may be obtained by placing two positive sources at (15.5,4.5,0) and (15,6,0) with fluxes 40 and 30, respectively, and four negative sources at (3.5,5,0), (9,13,0), (11,11,0) and (23,18,0) with fluxes -10, -10, -25 and -10, respectively. Figure 6a shows the normal component of this magnetic field in a plane just above the source plane. This field is not in flux balance since the net flux through the base is 15. To make this field physically realistic a source, N_∞, of flux -15 is placed at infinity without loss of generality.

An actual potential field containing a CC multiply-connected source pair can be obtained by placing three positive sources at (16.5,3.4,0), (15.5,4.55,0) and (15,6,0) with fluxes 45, 35 and 34, respectively, and four negative sources

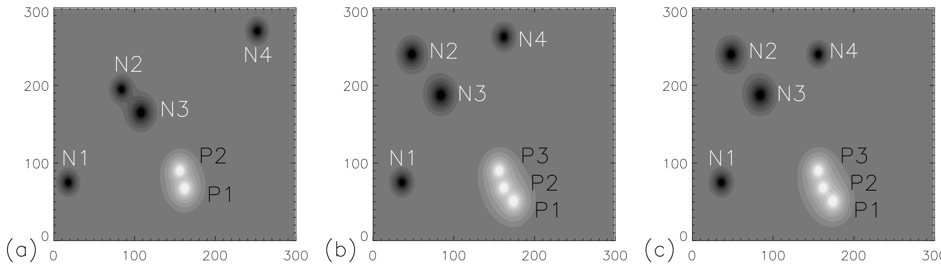

Figure 6. Contour plots of the normal flux distribution in $z = 0.01$ plane for the (a) OO, (b) CC and (c) OC multiply-connected source pairs. The contour levels in all figures are the same.

at (5,5,0), (6,16,0), (9,12.5,0) and (15.5,17.5) with fluxes -12, -22, -30 and -13, respectively. Again the net flux for this region is 37 and so without loss of generality a balancing source, N_∞ of strength -37 is sited at infinity. The normal component of this field just above the source plane is shown in Figure 6b. The main difference to the flux distribution in Figure 6a is the extra positive source in the bottom right-hand corner which makes the imbalance of flux even greater.

A potential field containing an OC multiply-connected source pair may be obtained by placing three positive sources at (16.5,3.4,0), (15.5,4.55,0) and (15,6,0) with fluxes 45, 35 and 34, respectively, and four negative sources at (5,5,0), (6,16,0), (9,12.5,0) and (15,16,0) with fluxes -12, -22, -30 and -10, respectively. The balancing source at infinity has strength -40. This is a very similar layout to that for the field containing the CC multiply-connected source pair except that the fourth negative source is weaker, and it has moved closer to N3. This results in its flux only connecting to the inner two positive sources (P2 and P3) and not to P1. The normal component of the field just above the source plane is shown in Figure 6c.

3.5. How common are multiply connected sources and nulls?

In the previous section we have merely given three examples of particular magnetic configurations that contain multiply-connected source and null pairs. Are these states generic or will a small perturbation in the position or flux of one of the sources destroy the multiply-connected nature of the topology? In an attempt to address this question we give two examples in which we vary the strength of one of the sources in one of the above configurations and determine the resulting magnetic topology.

The first case we consider has essentially the same distribution of magnetic sources as the OO multiply-connected setup shown in Figure 6a. There are two positive and five negative sources sited in the same positions as in

Figure 6a and they do not move. The fluxes of these sources are also the same as in Figure 6a except that the negative source N1 situated at (3.5,5,0) increases in flux from -25 to -1 and, in order to maintain overall flux balance, the source N_∞ decreases accordingly from 0 to -24.

There are four different topological states reached as the flux in N1 (and therefore N_∞) vary (Table I). In two of these topological states, accounting for two-thirds of the flux range considered, there is a OO multiply-connected source pair and a multiply-connected null pair. Indeed this state is still present when N1 has flux -25 resulting in flux balance without the need for a source at infinity. Clearly therefore topologies containing OO multiply-connected source pairs are generic.

Table I. Details of the different topological states reached due to seven sources (six when N1=-25) distributed in an identical manner and with the same fluxes as those in the OO-multiply-connected configuration in Figure 6a except here the flux of source N1, situated at (3.5,5,0), varies in strength from -25 to -1. Similarly the flux of source, N_∞, also varies accordingly to maintain overall flux balance. The number of separators X and the null pairs connected by separators are listed. The number of singly-connected D_1 and doubly-connected D_2 flux domains are stated as are the total number of flux domains $\mathcal{D}$ and the number of coronal domains D_c. The remaining $\mathcal{D}-D_c$ flux domains are ordinary.

N1 flux	X	Null pairs	D_1	D_2	$\mathcal{D}$	D_c
$-25(N_\infty = 0)$	4	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$	7	1 (OO)	9	1
$(-24.9999, -9.1754)^*$	4	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$	8	1 (OO)	10	1
$(-9.1754, -7.9658)$	3	$B1$-$A2$, $B1$-$A2$, $B1$-$A3$	7	1 (OO)	9	1
$(-7.9658, -1)$	2	$B1$-$A2$, $B1$-$A3$	8	0	8	0

To enable the reader to visualise the topologies in Table I it helps to look again at the cross-section in Figure 3. When the flux of N1 is in the range highlighted by a star in Table I the topological structure of the field is the same as that shown in Figure 3. However, when the flux of N1 lies in the range (-9.1754,-7.9658) source N1 is no longer strong enough to connect to P2 and all its flux is linked to P1, thus the separator $B1$-$A1$ is lost by way of a global separator bifurcation (Brown and Priest, 1999a). As the strength of N1 is reduced further, such that its flux lies in the range (-7.9658,-1), the influence of N4 increases and a second global separator bifurcation occurs. This results in the loss of the left-hand $B1$-$A2$ separator and left-hand P2-N2 domain.

Table II. Details of the different topological states reached due to 8 sources distributed in an identical manner and with the same fluxes as those in the CC-multiply-connected configuration in Figure 6b except here the flux of source P2, situated at (15.5,4.55,0), varies from 1 to 61. Similarly the flux of source, N_∞, also varies accordingly to maintain overall flux balance.

P2 flux	X	Null pairs	D_1	D_2	$\mathcal{D}$	D_c
$(1., 5.1990)$	4	$B1$-$A1$, $B1$-$A3$ $B2$-$A1$, $B2$-$A3$	11	0	11	1
$(5.1990, 24.5873)$	6	$B1$-$A1$, $B1$-$A3$ $B2$-$A1$, $B2$-$A2$, $B2$-$A2$, $B2$-$A3$	11	1 (OO)	13	2
$(24.5873, 35.1698)^*$	8	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A1$, $B2$-$A2$, $B2$-$A2$, $B2$-$A3$	11	2 (OO/CC)	15	4
$(35.1698, 35.2395)$	7	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A1$, $B2$-$A2$, $B2$-$A2$	10	2 (OO/OC)	14	3
$(35.2395, 37.6833)$	6	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A1$, $B2$-$A2$	11	1 (OC)	13	2
$(37.6833, 41.7958)$	8	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A1$, $B2$-$A2$, $B2$-$A4$, $B2$-$A4$	11	2 (OC/OO)	15	3
$(41.7958, 42.1640)$	7	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A2$, $B2$-$A4$, $B2$-$A4$	10	2 (OO/OO)	14	2
$(42.1640, 42.9614)$	6	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A4$, $B2$-$A4$	9	2 (OO/OO)	13	2
$(42.9614, 51.6945)$	5	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$ $B2$-$A4$	10	1 (OO)	12	1
$(51.6945, 59.5647)$	4	$B1$-$A1$, $B1$-$A2$, $B1$-$A2$, $B1$-$A3$	9	1 (OO)	11	1
$(59.5647, > 61.)$	2	$B1$-$A1$ $B1$-$A3$	9	0	9	0

In the second case considered the flux of a positive source from the CC multiply-connected source field shown in Figure 6b is varied. The positions of all the sources are held fixed as are their fluxes except for the flux of P2 and the flux of N_∞ which must increase in line with any increase in the flux of P2 to maintain overall flux balance. Table II details the topological changes in the separators and flux domains as the flux in P2 increases from 1 to 61. A total of eleven different magnetic configurations are found of which four have one multiply-connected source and null pair and five have two. The most common multiply-connected source pair is a OO pair with the CC being the least common. Note, there are always at least n coronal domains in a configurations with n multiply-connected source pairs regardless of whether the source pair is OO, OC or CC.

To visualise the topologies found in Table II we direct the reader back to Figure 4. The cross-section in this figure shows two solid convex curves marking the intersection of the separatrix-surfaces from the positive nulls B1 and B2 with this vertical plane. The area between these two solid curves contains the flux from P2. When P2 has a flux of 35, i.e., it has flux in the range highlighted by a star in Table II, then there are eight separators and the configuration is the same as that shown in Figure 4.

First let us consider what happens if we decrease the flux in P2 such that is lies in the range (5.1990,24.5873) (and hence decrease the strength of N_∞). P1 becomes more dominant, shrinking the outer solid curve, and forcing the inner solid curve containing the flux from P3 to also change. The flux domains P3-N1 and P3-N4 increase at the expense of the P3-N2 flux domain which disappears via a global double-separator bifurcation (Haynes et al., 2007), resulting in the loss of both $B1$-$A2$ separators. This destroys the CC multiply-connected source pair although the OO multiply-connected source pair still remain. A further decrease in the strength of P2 causes another global double-separator bifurcation in which the P2-N2 connection is destroyed as are both $B2$-$A2$ separators, leaving just 4 separators and no multiply-connected source pairs.

Now if we look at what happens if the flux in P2 increases above 35. Clearly this means that the area between the two solid curves in the cross-section seen in Figure 4 grows leading to the expansion of the outer solid curve. The first bifurcation resulting from the growth of P2 is a global separator bifurcation and results in the loss of the P1-N4 connection and the loss of the corresponding $B2$-$A3$ separator. This still leaves two multiply-connected source pairs, one OO and the other OC. The next bifurcation as the P2 flux continues to grow is also a global separator bifurcation which destroys the P1-N3 domain and one of the $B2$-$A2$ separators. This leads to the loss of the OO multiply-connected source pair.

The continued increase in the flux of P2, such that it lies in the range (37.6833,41.7958), results in an increase in the number of domains and separators: a global double-separator bifurcation occurs at which the separatrix surface from the B2 null intersects with the separatrix surface from the A4 null (largest dashed curve). This creates two new separators linking $B2$-$A4$, a new coronal flux domain P2-N_∞, and a new OO multiply-connected source pair P1-N2.

As P2 increases still further the $B2$-$A1$ separator is lost by a global separator bifurcation, as is the P1-N1 flux domain and the P2-N3 multiply-connected source pair changes from a OC to a OO pair. The next bifurcation, another global separator bifurcation, results in the loss of the $B2$-$A2$ separator and the P1-N3 flux domain, though the two multiply-connected source pairs still remain. Further increases in the P2 flux bring it into the range (42.9614,51.6945) and result in the loss of one of the $B2$-$A4$ separators via a global separator bifurcation and the loss of one of the P1-N2 flux domains, leaving just one multiply-connected source pair and one multiply-connected null pair. The final $B2$-$A4$ separator is lost as the P2 flux increases even more resulting in the separatrix surface from $B2$ completely encompassing the flux from P3 and the negative sources N1 to N4. This means that P1 is now only connected to N_∞. The final configuration considered is reached with a further increase in the strength of P2 and a global double-separator bifurcation which results in the loss of the two $B1$-$A2$ separators and reduces the P2-N3 source pair from multiply-connected to simply connected. It also destroys the coronal domain P3-N3. Not surprisingly, further increases in the flux of P2 above 61 result in both the creation and destruction of more separators, but we do not discuss these further bifurcations.

These two small case studies clearly show that multiply-connected source pairs are not only generic, but, moreover, are common in potential configurations with more than a handful of sources. If similar case studies had been performed in which particular sources were moved then similar conclusions are reached. Potential fields that produce higher-order multiply-connected null and source pairs are natural extensions to the three potential architectures described above. They may be created by adding extra sources or simply by varying the position and strength of the existing sources. Furthermore, magnetic topologies involving coronal nulls and/or upright nulls may also give rise to multiply-connected source pairs although we do not discuss these states in this paper.

4. Multiply-Connected Source Pairs in a Resistive MHD Experiment

In potential magnetic configurations the spines and fan surfaces are inclined at right angles at a null point (Parnell et al., 1996). Furthermore, all the separatrix surfaces from prone nulls are perpendicular to the source plane and encompass their respective source with relatively simple surfaces. This is not the case in an MHD regime where there may be electric currents and the plasma and magnetic field can interact with one another. Thus, in magnetic fields evolving in a resistive MHD (as opposed to a potential) manner, separatrix surfaces may ‘warp’ in complex ways (provided they do not cross themselves). This warping is a response to the motions of the plasma or to changes in total pressure due to the ideal nature of most of the plasma. Separatrix surfaces from opposite polarity nulls can cross each other. The crossing (or uncrossing) of two separatrix surfaces is known as a global bifurcation. The process creates new separators and extra flux domains, but no new nulls. The extra flux domains generally contain newly reconnected flux and, hence, a bifurcation involves the onset of new reconnection processes. If the separatrix surfaces uncross then separators and flux domains are lost and the bifurcation represents the demise of some existing reconnection process. Hence, bifurcations result from resistive MHD effects.

Since magnetic fields tend to expand out to fill a region, separatrix surfaces from nulls of opposite polarities naturally touch first above the source plane (in the corona). A build-up of the magnetic field in the vicinity of these touching surfaces naturally causes electric currents to develop in a variety of ways (see Priest and Forbes (2000) for examples). These currents usually grow to intense values if the surfaces continue to be brought together. In a resistive MHD situation reconnection will eventually kick in, regardless of the form of resistivity. This reconnection results in a global double-separator bifurcation which is the natural way in which the two separatrix surfaces may create an overlap if they meet inside a domain as opposed to at the boundary of the domain. A global double-separator bifurcation results in the creation of at least one coronal domain and at least one multiply-connected source pair, as well as a multiply-connected null pair.

It has been found by Haynes et al. (2007) that source pairs of both low and high multiplicity naturally occur in simple MHD configurations. Below we briefly show examples of how multiply-connected source pairs arise in the resistive MHD evolution of a simple magnetic field.

4.1. Doubly-Connected Source Pairs

The setup used by Haynes et al. (2007) is very simple and merely involved two opposite-polarity sources (P1 and N1) in a uniform overlying field, arising from two far-off sources (P2 and N2). Since the field is not potential a cross-section through the flux domain from one of the sources may take almost any form provided the separatrix surface does not cross itself (Figure 7). Initially, the sources P1 and N1 are unconnected and their sources are driven such that their flux domains are forced together leading to reconnection. The first interacting state involves an *OO doubly-connected source pair* (Figure 7). The multiply-connected source pair is P2-N2 and its flux is split into two domains: a domain surrounding the flux from two central source domains (P1-N2 and P2-N1) and a domain trapped below the newly formed coronal domain that holds flux connecting the two central sources (P1-N1) (Figure 7c).

There are just two nulls in the region on the source plane and these are multiply-connected by two separators (Figure 7d). A cross-sectional cut in a plane through these separators (Figure 7b (i)) clearly shows the five flux domains. Cross-sectional cuts parallel to this cut that do not intersect the separators are also shown (Figure 7b (ii) and (iii)). Here the overlying field (P2-N2) source pair must also be divided into two flux domains which are each separated by a separatrix surface; however, there is just one separatrix

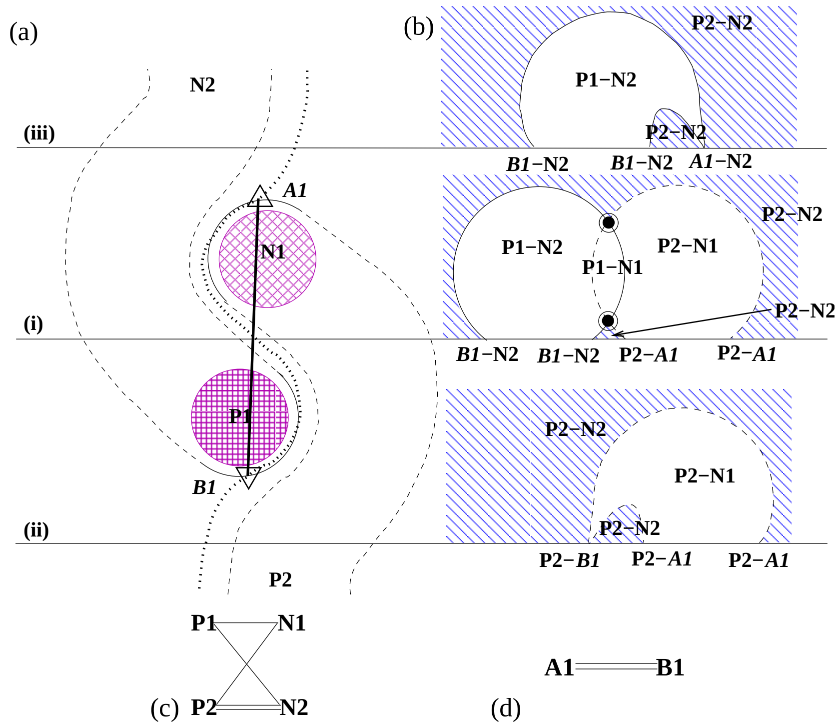

Figure 7. Sketches of (a) a footprint, (b) a vertical cross-section, (c) the domain and (d) null graphs of a non-force-free (MHD) magnetic field that has an OO multiply-connected source pair. Notation is the same as that in Figure 3

surface in each of these cuts. The separatrix surface encloses each of the three domains by doubling back on itself in the manner shown. The *B1*-N2 separatrix surface kisses the source plane along the spine *A1*-N2. Similarly in the other cross-section the P2-*B1* spine is part of the P2-*A1* separatrix surface. Thus, the P2-N2 field trapped below the coronal domain P1-N1 is an ordinary flux domain ringed by a separator above the source plane and whose entire volume above the source plane is surrounded by a separatrix surface.

The second type of doubly-connected source pair seen by Haynes et al. (2007) is an *OC source pair*. In fact, due to the symmetry in their setup, they find two such source pairs in the same magnetic configuration. This configuration (shown in Figure 8) contains three separators connecting the null pair (Figure 8d). These separators separate four domains which can be clearly seen in an arc around the central P1-N1 domain (Figure 3b). The four domains are in order from left to right, P2-N1, P1-N2, P2-N1 and P1-N2. The middle two are purely coronal domains whereas the other two are both ordinary.

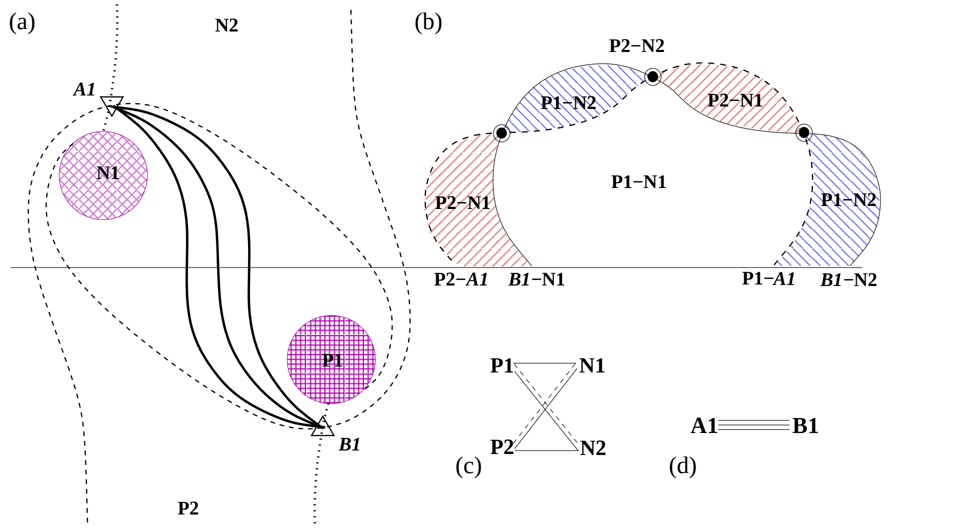

Figure 8. Sketches of (a) a foorptint, (b) a vertical cross-section, the (c) domain and (d) null graphs of a non-force-free (MHD) magnetic field that has two OC multiply-connected source pairs. Notation is the same as that in Figure 3

4.2. Triply-Connected Source Pair

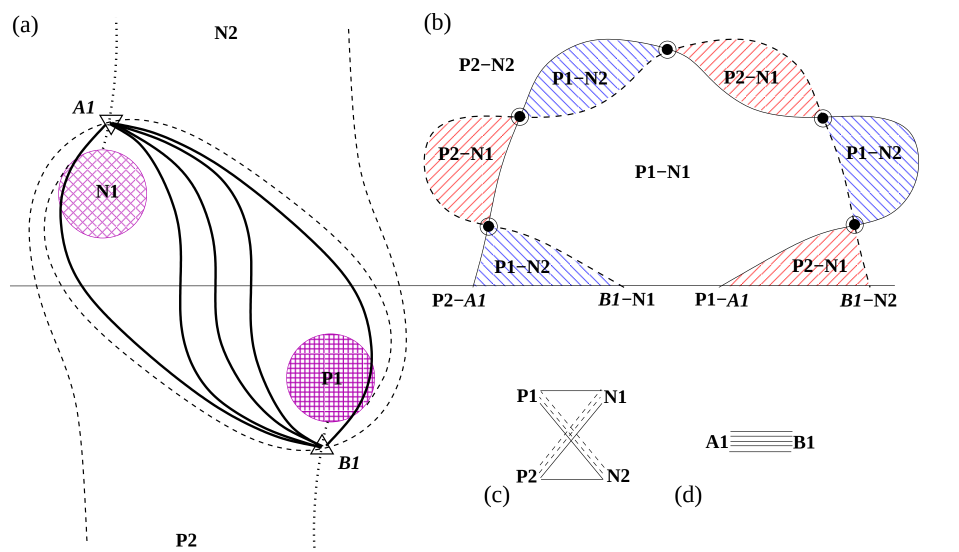

Figure 9. Sketches of (a) a footprint, (b) a vertical cross-section, the (c) domain and (d) null graphs of a non-force-free (MHD) magnetic field that has two OCC multiply-connected source pairs. Notation is the same as that in Figure 3

The experiment run by Haynes et al. (2007) evolved from the OO to OC multiply-connected state by way of a magnetic configuration with two triply-connected source pairs. This magnetic field was created by the bulging of the P1-N1 flux domain as it filled up with more and more flux. Firstly, the lower of the two separators in the OO multiply-connected state was lost by a global separator bifurcation which also saw the disappearance of the small trapped P2-N2 domain. Then, as the P1-N1 domain expanded further it eventually reached the outer walls (separatrix surfaces) of the P1-N2 and P2-N1 domains where it underwent a global double-separator bifurcation to create two source pairs with multiplicity three (Figure 9). These source pairs, P1-N2 and P2-N1 each have two coronal domains and one ordinary domain and so are OCC multiply-connected source pairs (Figure 9c). These domains are interspersed in an arc around the P1-N1 domain with five separators connecting the same pair of nulls (Figure 9b and 9d). The loss of the two lower separators by global separator bifurcations at the base enables the magnetic field to reduce to the topology shown in Figure 8 with two OC multiply-connected source pairs.

5. Importance of Multiply-Connected Source Pairs

We have presented examples of multiply-connected source pairs created in potential and non-potential magnetic fields associated with sources lying in a flat source plane. However, as has already been mentioned in the introduction, they can also be found in magnetic fields derived from sources lying on a spherical surface (Maclean et al., 2006). In their example a simple situation, involving only four sources, was found to create a multiply-connected source pair, suggesting that in a spherical geometry, such as the Sun, they may be even more common than in planar geometries. Furthermore, it is not difficult to imagine that quasi-separatrix surfaces will behave similarly and be able to cross multiple times creating multiply-connected quasi-separator pairs. Thus, it is not unreasonable to assume that multiply-connected source pairs will be common throughout the solar atmosphere. Below we therefore consider the implications of them and multiply-connected null pairs.

5.1. Implications of Multiply-Connected Source Pairs

The most obvious implication is that the number of flux domains is not the same as the number of connected source pairs, as mentioned by Longcope and Klapper (2002). Thus finding all the flux domains of a magnetic configuration is not necessarily trivial. As Close, Parnell, and Priest (2005b), Barnes, Longcope, and Leka (2005) and Haynes et al. (2007) have all shown, the Beveridge-Longcope equation is very useful in determining the number of flux domains provided the number of separators, the number of coronal nulls and the total number of sources are all known. To determine which source pairs are multiply connected and their multiplicity is, however, not possible from this equation.

Multiply-connected source pairs are always associated with multiply-connected null pairs (multiple separators). So in magnetic fields with multiply-connected source pairs there will be an above-average number of separators. Furthermore, separator reconnection has been found to be one of the most significant types of three-dimensional reconnection (Longcope, 1998; Galsgaard, Parnell, and Blaizot, 2000; Longcope, 2001; Parnell and Galsgaard, 2004; Galsgaard and Parnell, 2005; Priest, Longcope, and Heyvaerts, 2005; Priest and Titov, 1996; Longcope, 2005; Haynes et al., 2007). Thus, it is likely that the more separators there are the quicker and more wide-spread the release of magnetic energy will be.

Also Haynes et al. (2007) found that a global double-separator bifurcation produced two separators along which reconnection occurred in an opposite sense. For instance, consider Figure 7 which shows a magnetic configuration seen in the Haynes' experiment. It involves two separators that were created by a global double-separator bifurcation along with a new coronal domain P1-N1. Haynes et al. (2007) found that the reconnection along the upper separator created new P1-N1 and P2-N2 flux from original P1-N2 and P2-N1 flux. However, along the lower separator P1-N1 and P2-N2 flux was reconnected creating more P1-N2 and P2-N1 flux. Hence, there is the possibility of flux undergoing the same reconnection process more than once. The implications on the properties of the plasma of this type of behaviour are currently not well understood, but it is possible that it may lead to enhanced or more prolonged heating of certain flux loops (Parnell, Haynes, and Galsgaard, 2007).

One striking characteristic of the potential fields with multiply-connected source pairs is that these magnetic topologies all require clustering of like-polarity fragments to produce the multiple connections. This does not mean there cannot be any mixing of polarities, but that clustering is important in potential fields. Therefore, magnetic fields around active regions are likely to be ideal sites for multiply-connected source pairs, as demonstrated by Barnes, Longcope, and Leka (2005). Clustering of like-polarities is not required to produce multiply-connected source pairs and multiply-connected null pairs in resistive MHD fields.

5.2. Prediction of Multiply-connected Source Pairs

In certain situations it may prove easier to find all the separators in a region than to determine whether all the flux connecting a source pair is singly or multiply-connected. Thus we consider here whether it is possible to determine, without finding all the flux domains, which source pairs are multiply-connected and what their multiplicity is. For instance, if we consider the potential field discussed in Section 3.2, there are two options with 13 connected source pairs and $\mathcal{D} = 15$ that could satisfy the Beveridge-Longcope equation: namely having $D_1 = 12$ and $D_3 = 1$ or $D_1 = 11$ and $D_2 = 2$. The latter turns out to be the correct answer, but is it possible to obtain the former configuration? To create a triply-connected domain the separatrix surface from $B2$ would need to be next to, rather than encompassing the separatrix surface of $B1$. The surface from $B1$ would again have the same four separators, but then $B2$ would also have four separators to give 15 domains. This could be achieved if it had two doubly-connected null pairs $B2$-$A3$ and $B2$-$A2$, which would then give a total of fifteen domains including one triply-connected source pair P1-N3. However, it would also have three multiply-connected null pairs as opposed to two and have only twelve connected source pairs rather than thirteen. This is because it would also have a doubly-connected source pair P1-N4.

The above example suggests that, if one knew the number of multiply-connected null pairs, their multiplicity, as well as the number of sources, connected sources and coronal nulls, then it may be possible to determine the number of multiply-connected domains of a particular multiplicity. For some configurations this is the case, but not for others, as is shown by the following example.

Consider the magnetic field described in Section 4.2. This field involves four sources: two positive and two negative and one pair of nulls: one positive and one negative. There are five separators connecting this pair of nulls hence, from Equation 2,

$$\mathcal{D} = S + X + N_c - 1 = 4 + 5 + 0 - 1 = 8.$$

All the source pairs are connected so $\sum_n D_n = 4$. One possibility is that $D_1 = 2$, $D_2 = 0$ and $D_3 = 2$. This magnetic configuration is realisable and is shown in Figure 9. Another possibility that is also realisable is $D_1 = 0$ and $D_2 = 4$ (Figure 10). Here, all connected source pairs have both an ordinary and a coronal flux domain. Both of these two configurations have a total of four ordinary and four coronal domains and therefore the numbers of each type of domain cannot be used to distinguish between these fields either. The only way is to actually calculate the magnetic field and count flux domains in each case.

The latter configuration with four doubly connected source pairs can be simply obtained from the magnetic configuration shown in Figure 8. It arises if there are strong outflows from the two outer separators of Figure 8.

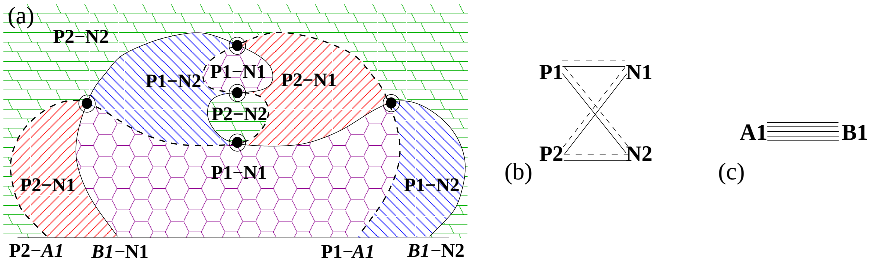

Figure 10. (a) Sketches of a vertical cross-section, (b) domain and (c) null graphs of a non-potential (MHD) magnetic field with four OC multiply-connected source pairs. Notation is the same as that in Figure 3.

6. Conclusions

Multiply-connected source pairs and multiply-connected null pairs can occur in magnetic fields, both force-free and non-force-free. The former are pairs of sources between which there is more than one flux domain and these go hand-in-hand with pairs of nulls that are connected by more than one separator. In such magnetic fields there are (i) more possible sites for reconnection, (ii) hence magnetic energy may well be released faster and (iii) flux can be reconnected multiple times between the same two domains.

The existence of multiply-connected source and null pairs means that the number of flux domains in a magnetic topology is not equal to the number of connected source pairs. The Beveridge-Longcope equation is useful in revealing if source pairs are multiply connected, but it cannot in general reveal the numbers or type of multiply-connected source pairs.

Multiply-connected null pairs are generally created by the global double-separator bifurcation. Such a bifurcation is the most natural type of global bifurcation in non-force-free fields and hence multiply-connected null and source pairs are likely to be common in complex magnetic fields. Flux domains linking a multiply-connected source pair can be either ordinary, coronal or semi-coronal, but not isolated, since such source pairs are related to separators.

Bifurcations involve changes in topology as a result of 3D reconnection. They occur where strong current sheets have formed and hence there are several possible ways in which they could occur, including MHD instabilities, motions due to magnetic pressure and/or tension, and rippling of the separatrix surface by MHD motions (Priest and Forbes, 2000).

Separatrix surfaces from potential null points tend to be relatively simple in shape. However, separatrix surfaces from a pair of generic opposite-polarity prone nulls may intersect not just twice, but three or more times. Separatrix surfaces in non-force-free fields are more complex than potential fields and may be warped in complicated ways and so the multiplicity of MHD null pairs can become much higher.

It would be useful to be able to predict the number and multiplicity of multiply-connected source pairs from information on the number and type of each element in the skeleton such as sources, domains and separators. For many simple potential magnetic fields this is possible, but for more complex potential fields and even for simple non-force-free fields it is not possible as we have demonstrated here via a simple example. Thus, there seems to be no easy way to predict the true nature of a magnetic field without actually calculating and then finding all the flux domains or fan sectors.

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