Solar Interior & Helioseismology

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STFC Introductory School, August 2016
The Unseen Interior

“At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden beneath substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?”

Pulsation opens a window!

“Ordinary stars must be viewed respectfully like objects in glass cases in museums; our fingers are itching to pinch them and test their resilience. Pulsating stars are like those fascinating models in the Science Museum provided with a button which can be pressed to set the machinery in motion. To be able to see the machinery of a star throbbing with activity is most instructive for the development of our knowledge.”

NSO Workshop #27
FIFTY YEARS OF SEISMOLOGY OF THE SUN & STARS
May 6-10, 2013 • Tucson, AZ, USA
Kepler Asteroseismic Science Consortium (KASC)

NASA Kepler Mission
Overview

- What are resonant oscillations of the Sun?
- How do we observe the oscillations?
- What can we learn from study of the oscillations?
  - Global helioseismology
  - Local helioseismology
- Bit of Asteroseismology
Sound waves generated at top of Convection Zone...

- Convection Zone
- Radiative Interior
- Photosphere

Acoustic source
The Resonant Sun

The Sun resonates like a musical instrument...

The organ pipe is the best analogue
Refraction of inward-travelling waves

Waves refract if launched inward at angle to radial direction:

\[ c \propto T^{1/2} \]

[c: sound speed; T: temperature]

End a of wave front at higher temp than b; so c there is higher!
Trajectories of acoustic waves in interior

Courtesy J. Christensen-Dalsgaard
Trajectories of acoustic waves in interior

Waves launched at steeper angle to radial direction penetrate more deeply!
Trajectories of acoustic waves in interior

These more-deeply penetrating waves have longer horizontal wavelengths: bigger skip distance goes with larger $\lambda_h$ [smaller $k_h$]

Key property: probe different depths

Courtesy J. Christensen-Dalsgaard
Standing acoustic wave patterns...

Internal acoustic ray paths

Surface displacement: oscillation patterns in 3D

red waves give...
blue waves give...
Dispersion relation involving wave frequency ($\omega$) and wavenumber ($k$)

- Simple $\omega = ck$ relation modified:
  - Interior stratified under gravity
  - Total internal reflection implies existence of cut-off frequency
  - Radial ($r$) and horizontal ($h$) wave numbers required
Dispersion relation

- Allow for different types of internal wave:
  - Acoustic waves: compression dominates
  - Buoyancy waves: displacement dominates
Dispersion relation

Simple $\omega = ck$ relation modified to:

$$k_r^2 = \frac{\omega^2 - \omega_{ac}^2}{c^2} + \frac{k_h^2 (N^2 - \omega^2)}{\omega^2}$$

$\omega_{ac}$: acoustic cut-off frequency (pressure – p-modes)

$N$: Brunt Väisälä frequency (characterises oscillation of fluid element displaced from rest position) (gravity – g-modes)

Both characteristic frequencies are a function of location within Sun

Where $k_r$ is positive, solution is oscillatory
Categorization of modes

- Spatial part can be described by spherical harmonic functions
- Spherical harmonic integers, $l$ and $m$, describe spatial pattern
- $m$ is related to rotation (more later)
- Radial order $n$ corresponds to the number of nodes in the radial direction
Categorization of modes

Angular degree, \( l \), depends on horizontal wave number and outer radius of cavity according to:

\[
k_h = \frac{2\pi}{\lambda_h} = \frac{L}{R} = \frac{\sqrt{l(l+1)}}{R}
\]

So:

\[
\lambda_h = \frac{2\pi}{k_h} = \frac{2\pi R}{L}
\]
The Resonant Sun
Analogy of a musical instrument...

The Wilhelmy American Flag Glass Pipe Organ
Condition for constructive interference

- Take a simple 1-D pipe
  - Pipe runs from $z=0$ to $z=L$
- Condition for standing waves depends on boundary conditions
  - *i.e.*, is pipe open or closed?
Resonance in simple 1-D pipes
The frequency spectrum

- Fully open length $L$
- Semi-closed length $L/2$
Fully open pipe

Fundamental (1\textsuperscript{st} harmonic): \[ L = \frac{\lambda}{2} \]

1\textsuperscript{st} overtone (2\textsuperscript{nd} harmonic): \[ L = \lambda \]

2\textsuperscript{nd} overtone (3\textsuperscript{rd} harmonic): \[ L = \frac{3\lambda}{2} \]

So:
\[ L = \left( \frac{n+1}{2} \right) \lambda. \]

\( n = 0, 1, 2 \ldots \) etc.
Fully open pipe

Then:

$$k_z L = \frac{2\pi}{\lambda} \left( \frac{n + 1}{2} \right) \lambda,$$

where $k_z$ is wave number:

$$\therefore k_z L = \int_L k_z \, dz = (n + 1) \pi.$$
Fully open pipe

- This is classic interference condition, i.e.,

\[\Delta \phi = \int_{L} k_z dz = (n + \alpha) \pi,\]

where \(\alpha\) is a constant.

- Clearly \(\alpha = 1\) for fully open pipe
- For semi-closed pipe, \(\alpha = \frac{1}{2}\)
But hang on! Sun is 3-D body

- Can progress through 2 dimensions and then to 3 via a cone.
- Get rich spectrum of modes
- Basic ideas still hold
Interference condition in Sun

- We can write the same interference condition for waves trapped inside the Sun.
- We have: integral of radial wave number between lower \((r_t)\) and upper \((R_t)\) turning points, \(i.e.,\)

\[
\Delta \phi = \int_{r_t}^{R_t} k_r dr = (n + \alpha) \pi.
\]
Interference condition in Sun

- Value of $\alpha$ depends on boundary conditions at lower and upper turning points
- Need to consider each turning point separately, so two different contributions to $\alpha$
- It turns out that $\alpha \approx 1.5$
The frequency spectrum

- Each transverse solution has its own set of overtones
- So, for Sun (with spherical harmonic solutions giving transverse part):
  - each angular degree, $l$, has its own family of overtones (described by order $n$)
- Doppler velocity or Intensity
Spherical Harmonics

$l=2, m=2$  $l=4, m=0$

$l=20, m=17$  $l=20, m=0$

$l=20, m=20$
Pulsation Timescale

- Fundamental period of radial pulsation:
  \[ \Pi \propto \left\langle \rho \right\rangle^{-1/2} \]
  Ritter 1880; Shapley, 1914

- Estimate period from sound crossing time

- Period similar to dynamical timescale ('free fall' time)
Pulsation Timescale

Sun: fundamental radial mode period

\[ \Pi_f \approx 1 \text{ hour} \]

\[ \nu_f \approx 250 \mu \text{Hz} \]

Courtesy D. Hathaway
Standing acoustic wave patterns...

Internal acoustic ray paths

Surface displacement: oscillation patterns in 3D

red waves give...

blue waves give...
Frequency spectrum of low-degree (low-\(l\)) modes (contains overtones of \(0 \leq l \leq 3\))

![Graph showing frequency spectrum with high-overtone \((n \approx 20)\) modes highlighted.]

High-overtone \((n \approx 20)\) modes!

250 \(\mu\text{Hz}\)

BiSON data
Standing acoustic wave patterns...

Internal acoustic ray paths

Surface displacement: oscillation patterns in 3D

red waves give...

blue waves give...
Frequency spectrum: $l$-$\nu$ diagram

Consider the overtones of each degree, $l$
Frequency spectrum: $l$-$\nu$ diagram

Data collected by MDI instrument on board SOHO
Sun-as-a-star observations

BiSON
6 stations

VIRGO/SPM
Birmingham Solar Oscillations Network ≡ BiSON
Measures Doppler Velocity
Very high precision
http://bison.ph.bham.ac.uk/
Resolved-Sun Observations

SOI/MDI

GONG
GONG 6 stations

VIRGO/LOI
Resolved-Sun Observations

HINODE  HMI/SDO  PICARD
Global and local helioseismology

- Global seismology:
  - Constituent waves live long enough to travel round the Sun
  - Modes give longitudinal average of properties (also cannot distinguish asymmetry in properties above and below equator)
Global and local helioseismology

- Local seismology:
  - Do not wait for resonance to establish globally
  - Observe effects of interference in local volumes beneath surface
Frequency spectrum: $l-\nu$ diagram

Data collected by MDI instrument on board SOHO

Horizontal wavenumber / angular degree
Local methods: rings and trumpets

Resolve into orthogonal horizontal wave numbers (angular degrees, $l$)

2-D $l$-$\nu$ diagram becomes series of nested 3-D surfaces

What were ridges are now flaring ‘trumpets’)
Local methods: rings and trumpets

Take cut at fixed frequency: get series of rings

Analysis of rings can be used to measure flows, fields *etc.*, which distort shapes of rings

Measure properties beneath small patches on surfaces, *e.g.*, beneath active regions

Courtesy D. A. Haber and collaborators
Flows and wave speed variation beneath sunspot (from local methods)

Arrows show flows:
- Larger ➔
- Smaller ➔

Colours show wave-speed:
- Faster… in red
- Slower… in blue

Courtesy A. G. Kosovichev, SOI Stanford
Solar Sub-Surface Weather (Local methods)

Flows (arrows) beneath regions of magnetic flux (red)

Measure flows underneath small patches

Rotation brings new patches into view

Build up strips, side-by-side, in longitude

Courtesy D. A. Haber and collaborators
Local methods: time-distance seismology

Analogue of terrestrial time-distance methods

Measure time taken for waves to reach detectors from natural, or man-made, seismic events

Use this information to infer internal properties

Or to locate sources
Local methods: time-distance helioseismology

Can try something similar on Sun, but:

- We cannot create our own seismic events
- Seismic generation of waves takes place at multitudinous locations across surface of Sun!
- Use cross-correlation techniques

Courtesy A. G. Kosovichev, SOI Stanford
Local methods: time-distance helioseismology

To first order, waves launched at given angle take same time to reappear at surface

This is so-called single-skip time

Courtesy A. G. Kosovichev, SOI Stanford
Local methods: time-distance helioseismology

- Take two locations on surface
- Measure Doppler velocity or intensity at these locations
  - Separation will correspond to skip distance for waves launched at particular angle
  - Signals will be strongly correlated at separation in time corresponding to time to make single skip
  - Can also pick up time to make two skips… and three
Local methods: time-distance helioseismology
Local methods: time-distance helioseismology

- In practice, cross correlated between central patch and surrounding annulus
- Search for strong correlations at each separation
- Can build solar equivalent of terrestrial time-distance plot
- Infer internal properties from travel-time information
Space weather predictions

Far-side imaging of active regions
Heinrich Samuel Schwabe found the solar cycle in 1843 for which the RAS gave him a gold medal.

Carrington measured the position of spots and set up the notion of Carrington rotations. His work halted in ~1861 when he had to go back to run the family business. (Brewing)
- 9\textsuperscript{th} May 2016
- Now fairly quiet
- Black dot lower left is Mercury
Maximum has two humps

The maximum is smallest for 80 years and the activity is much like it was around 100 years ago when the Sun went through a quiet period.
Clear correlations with surface measures of activity – early data

frequency shifts vs. time correlated with 10.7 cm radio flux

- **Black** are frequency shifts (overlapping)
- **Red** is 10.7 flux
- **Green** is used in search for short-term fluctuations

Broomhall et al., 2009
Rotational Splitting

Axis of rotation
Rotational Splitting

Shifted frequencies: \( \omega_m \approx \omega_0 + m\Omega \)

\( \Omega \) is suitable average of position dependant angular velocity in cavity probed by mode
(Note: correction from coriolis force is small for Sun.)

Approximate magnitude of solar rotational splitting:
\[
\Delta \nu_{\Delta l m l=1} = \frac{\Delta \omega_{\Delta l m l=1}}{2\pi} = \frac{\Omega}{2\pi} \approx 0.4 \mu \text{Hz}
\]
Power spectrum of low-\(l\) modes, showing rotationally split components

BiSON data

Frequency (micro-Hertz)

0.4 \(\mu\)Hz
GONG data

Internal Solar Rotation

Tachocline believed to be important for the generation of the varying magnetic field (but)
Slow Rotation of the Deep Interior!

Chaplin et al., 1999, MNRAS

Rotational frequency

$\Omega/2\pi$ (nHz)

Fractional radius

Period of 26.6 days

24.6 d

31.3 d

Chaplin et al., 1999, MNRAS
Rotation–activity relationship diagram for partly and fully convective stars – magnetic field without a tachocline?

The solar abundance problem

- Accurate chemical abundances vital for addressing many astrophysical problems
- Recent downward revision of solar “heavy element” abundance
  - Impact on solar models...
The solar abundance problem

Fractional sound speed difference
(observed minus model)

Fractional radius in solar interior

Older abundances
Newer abundances

Courtesy S. Basu
The solar abundance problem

- Various solutions proposed...
- Was thought problems localized in outer parts of Sun... **BUT**

Problems extend all the way to the core
Kepler Asteroseismic Science Consortium (KASC)

NASA Kepler Mission
Kepler-21

Star a bit more massive than the Sun and a bit more evolved. Has a rocky planet 1.5-times size of the Earth orbiting every 2.8 days at a distance of 4.3% of the Earth-Sun distance.

How will our Sun evolve?

As stars age, they increase in size.

Kepler sequence of 1 solar-mass stars.

Increasing size, age.
A key transition point in the evolution of a star is when it starts to burn Helium deep in its core.

- Very hard to determine from the surface.

Tracks show different masses (not following colours in scale)

Picture from Andrea Miglio
Can seismology give the answer?
Red Giant Branch or Helium core-burning?

- Compare structure in the acoustic spectrum……
Red Giants as observed by Kepler

- One of the great success stories of mission
- Detected ‘mixed’ modes which are sensitive to both the core and the outer regions (Beck et al. *Science* 2011 *332*)
- Have been able to detect evolutionary state of stars – have they reached helium-core burning yet (Bedding et al. *Nature* 2011 *471*)
- Measure rotation and able to show centre rotating much faster than surface – unlike the Sun (Beck et al. *Nature* 2012 *481*)
Very exciting times as we try to understand what happens inside stars
Thank you