# Kinematic MHD Models of Collapsing Magnetic Traps: Extension to 3D

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**Abstract.** We show how fully 3D kinematic MHD models of collapsing magnetic traps (CMTs) can be constructed, thus extending previous work on 2D trap models. CMTs are thought to form in the relaxing magnetic field lines in solar flares and it has been proposed that they play an important role in the acceleration of high-energy particles. This work is a first step to understanding the physics of CMTs better.

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## INTRODUCTION

We discuss the collapsing magnetic trap (CMT) model that has been suggested to form by the reconnecting coronal loops in a flare. It is thought that CMTs take part in flare particle acceleration [e.g. 1–3]. CMTs have also been studied as an acceleration method for particles in the Earth's magnetotail [e.g. 4, 5]. After being accelerated by the reconnecting current sheet, particles stream down the newly reconnected field lines and are then reflected either at a shock front or in the converging magnetic field in the lower corona (see figure in [1]). The post-reconnection field lines relax downwards so the particles become trapped in the collapsing field lines. In the following sections the theory developed by Giuliani et al. [3] is extended by the addition of a shearing velocity – as is expected to occur in flares – and a more general 3D theory is developed.

## 2.5D MODEL WITH SHEARING

We build on the model used by Giuliani et al. [3]. This was a 2.5D model using ideal kinematic MHD using only Ohm's law,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \tag{1}$$

and Faraday's law,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}. \tag{2}$$

The magnetic field may be written in the form

$$\mathbf{B} = \nabla A \times \mathbf{e}_z + B_z \mathbf{e}_z \tag{3}$$

A velocity field in the $x$ and $y$ directions must be given. The $z$ direction is invariant and it is assumed that $v_z = 0$, making the model 2.5D,

$$\mathbf{v} = (v_x, v_y, 0). \tag{4}$$

From these the equation for the propagation of the flux function becomes

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \mathbf{v} \cdot \nabla A = 0 \tag{5}$$

and the induction equation

$$\frac{\partial B_z}{\partial t} + \nabla \cdot (\mathbf{v} B_z) = 0 \tag{6}$$

Equations (5) and (6) express the conservation of magnetic flux. In the case without shear velocity $v_z$ the flux $\int B_z dxdy$ is conserved independently. To solve these equations for the time-dependent electromagnetic fields, a transformation between Lagrangian and Eulerian coordinates is used:

$$X = X(x,y,t), \qquad Y = Y(x,y,t). \tag{7}$$

The solution for the magnetic flux function $A(x,y,t)$ is then given by

$$A(x,y,t) = A_0(X(x,y,t), Y(x,y,t)) \tag{8}$$

where $A_0(X,Y)$ is the initial flux function. The magnetic field in the $z$ direction can be calculated by

$$B_z(x,y,t) = |J|^{-1} B_z(X(x,y,t), Y(x,y,t)) \tag{9}$$

where $J$ is the Jacobian determinant of the transformation between the Lagrangian and Euerian coordinates.

The downflow from the reconnected field lines relaxing produces a strong electric field in the central parts of the trap. This causes the charged particles to accelerate.

Using the fields that have been calculated, the particle orbits have also been calculated using the guiding centre approximation [see 6].

We can now add a shearing velocity. This is a non-zero $v_z$, which adds an initial twist to the field lines, allowing them to release more energy as they deshear as the trap collapses. This adds a source term to equation (6) to become

$$\frac{\partial B_z}{\partial t} + \nabla \cdot (\mathbf{v} B_z) = \nabla \cdot (v_z \mathbf{B}). \tag{10}$$

Where $B_z$ is the magnetic field in the $z$ direction, $\mathbf{v}$ the velocity field as defined in Eq. (4) and $\mathbf{B} = (B_x, B_y, 0)$. It turns out that this extended set of equations can be solved in a similar way to the case with $v_z = 0$ by adding a transformation for the $z$-coordinate of the form

$$Z = z + \bar{Z}(x,y,t). \tag{11}$$

The solution for the flux function remains the same, but the solution for $B_z$ becomes more complicated, because a non-zero $v_z$ allows $B_x$ and $B_y$ to by turned into $B_z$ and vice versa. The equation for $B_z$ is the same as in the 3D case below, see equation (13).

## 3D THEORY

To generalise the theory to 3D we can use a similar approach. To ensure the solenoidal condition, $\nabla \cdot \mathbf{B} = 0$, applies to the system we use Euler Potentials (Clebsch variables [see 7]):

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

There are other possible choices for the formulation of the magnetic field but this method was chosen because the equations for the Euler potential are analogous to the equation for the flux function in 2D. This method is suggested as an extension by Giuliani et al. [3].

The equations for the Euler Potentials can be written as

$$\frac{\partial\alpha}{\partial t} + \mathbf{v} \cdot \nabla\alpha = 0 \qquad \text{and} \qquad \frac{\partial\beta}{\partial t} + \mathbf{v} \cdot \nabla\beta = 0.$$

This makes the magnetic field evolve with the induction equation.

As before, the velocity is defined as by a transformation between Eulerian and Lagrangian coordinates.

$$X = X(x,y,z,t), \qquad Y = Y(x,y,z,t), \qquad Z = Z(x,y,z,t),$$

which are combined into the vector $\mathbf{X} = (X,Y,Z)$ for ease of reference. This is related to the velocity by the equation

$$\frac{d\mathbf{X}}{dt} = \frac{\partial\mathbf{X}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{X} = 0. \tag{12}$$

Solving this equation for $\mathbf{v}$ gives the velocity field for the system and all the other quantities follow from this. This system has been chosen so that equations (5) and (12) are similar thus the solution in this case can be found in a way analogous to above.

We can combine the initial magnetic field (in terms of the transformation) and the transformation to determine the magnetic field at any time,

$$\begin{aligned} B_z \;=\; & \left(\frac{\partial Y}{\partial x}\frac{\partial Z}{\partial y} - \frac{\partial Z}{\partial x}\frac{\partial Y}{\partial y}\right) B_{0x}(\mathbf{X}) \\ & + \left(\frac{\partial Z}{\partial x}\frac{\partial X}{\partial y} - \frac{\partial X}{\partial x}\frac{\partial Z}{\partial y}\right) B_{0y}(\mathbf{X}) \\ & + \left(\frac{\partial X}{\partial x}\frac{\partial Y}{\partial y} - \frac{\partial Y}{\partial x}\frac{\partial X}{\partial y}\right) B_{0z}(\mathbf{X}), \end{aligned} \tag{13}$$

where $\mathbf{B_0} = (B_{0x}, B_{0y}, B_{0z})$ is the initial magnetic field. We only give the expression for $B_z$ here, but $B_x$ and $B_y$ are given by similar formulae with the roles of $x$, $y$ and $z$ cyclically permuted. From the velocity and the magnetic field we can then compute the electric field and any other variables needed.

Figure 1 shows an example of magnetic field lines in a 3D collapsing trap model at two different times. In this case the initial magnetic field is a magnetic bipole generated by two point charges. This clearly shows the collapsing motion of the trap.

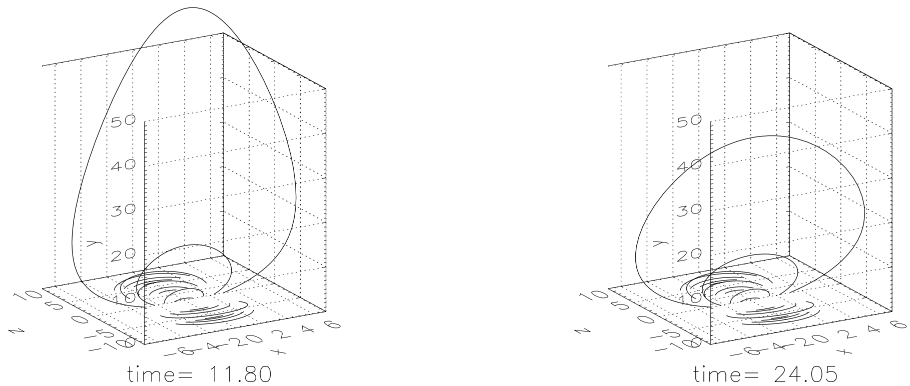


**FIGURE 1.** An example showing the magnetic field lines in a trap collapsing as time progresses.

# DISCUSSION AND CONCLUSIONS

We have developed a fully analytical model for kinematic time-dependent, 3D collapsing magnetic traps. Although this kinematic approach allows us full control over all the features of the model, the modelling of the plasma system is not self-consistent.

Studies are currently underway to find out what energy gains are possible for these 2.5 and 3D models. This involves looking into more realistic CMT models and calculation of the resulting particle orbits. From these distribution functions could be produced and compared with observations of particle motions in flares. Once models with reasonable energy gains have been found the model could be improved by the inclusion of Coulomb collisions, which would have a slight decelerating effect.

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