A Topological Analysis of the Magnetic Breakout Model for an Eruptive Solar Flare

Rhona Maclean[1], *Colin Beveridge*[2], *Dana Longcope*[2], *Daniel Brown*[1] *and Eric Priest*[1]
[1]UNIVERSITY OF ST ANDREWS AND [2]MONTANA STATE UNIVERSITY

ABSTRACT

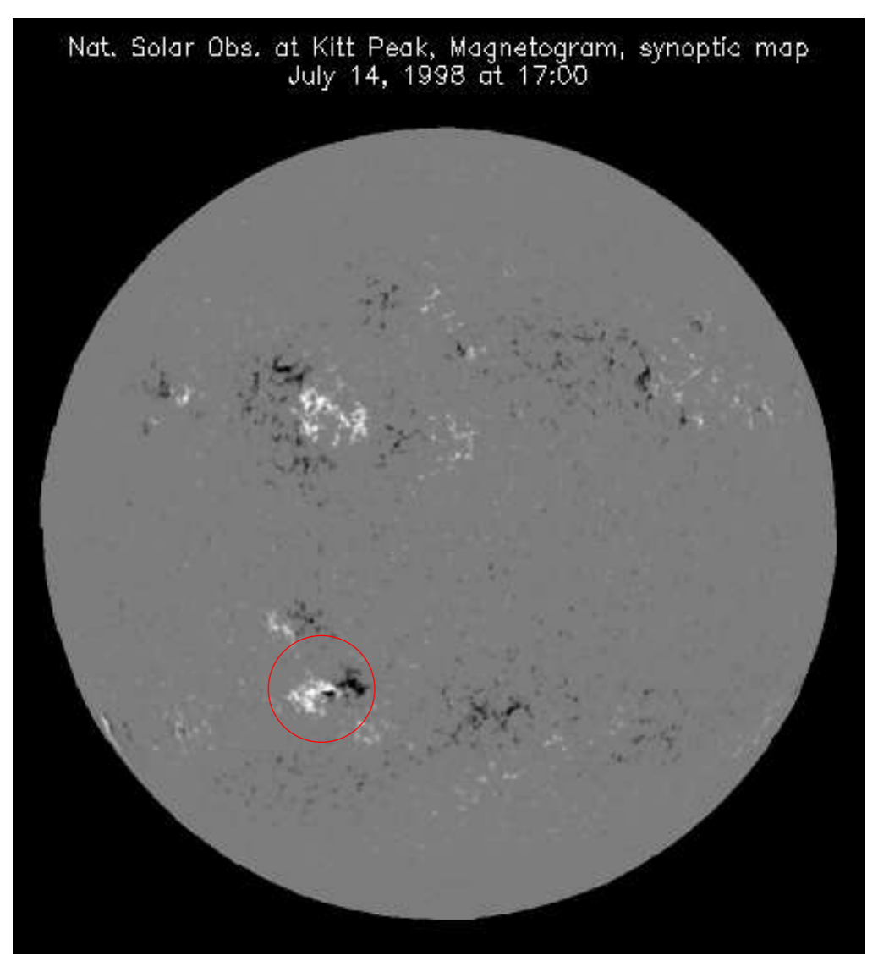
The magnetic breakout model gives an elegant explanation for the onset of an eruptive solar flare. In it, a flux system is initially enclosed by an overlying magnetic arcade. Shear is applied at a neutral line in the photosphere, causing reconnection at a coronal null point. This reduces the overlying flux and allows the enclosed flux to "break out" to large distances. We show here that any new class of field lines, such as those connecting to large distances, must be created through a global topological bifurcation. General rules are derived to predict the topological reconfiguration due to bifurcations. Topological analysis of a simple delta sunspot model reveals such global bifurcations, some leading to breakout, in evolutions of both potential and linear force-free fields.

1) MAGNETIC BREAKOUT

The magnetic breakout model is an explanation for the onset of a solar flare, proposed by Antiochos, DeVore and Klimchuk(1999). It consists of the following sequence of events:

- A central flux system is initially enclosed by an overlying arcade.
- Shear is applied near a neutral line in the photosphere, causing magnetic reconnection to take place in the vicinity of a magnetic null point in the corona.
- This weakens the overlying field and allows the originally enclosed flux to "break out" explosively.

Further work by Antiochos(1998) showed that the simplest configuration with sufficient complexity to allow this behaviour is the *delta sunspot*. A delta sunspot consists of two opposite-polarity sunspot umbrae contained within a common penumbra. Indeed, such a configuration is observed to be a prolific producer of flares (Tanaka, 1991). A sample magnetogram of a delta spot (circled in red) is shown in the diagram.



Magnetogram showing a delta sunspot

2) MODEL OF DELTA SUNSPOT

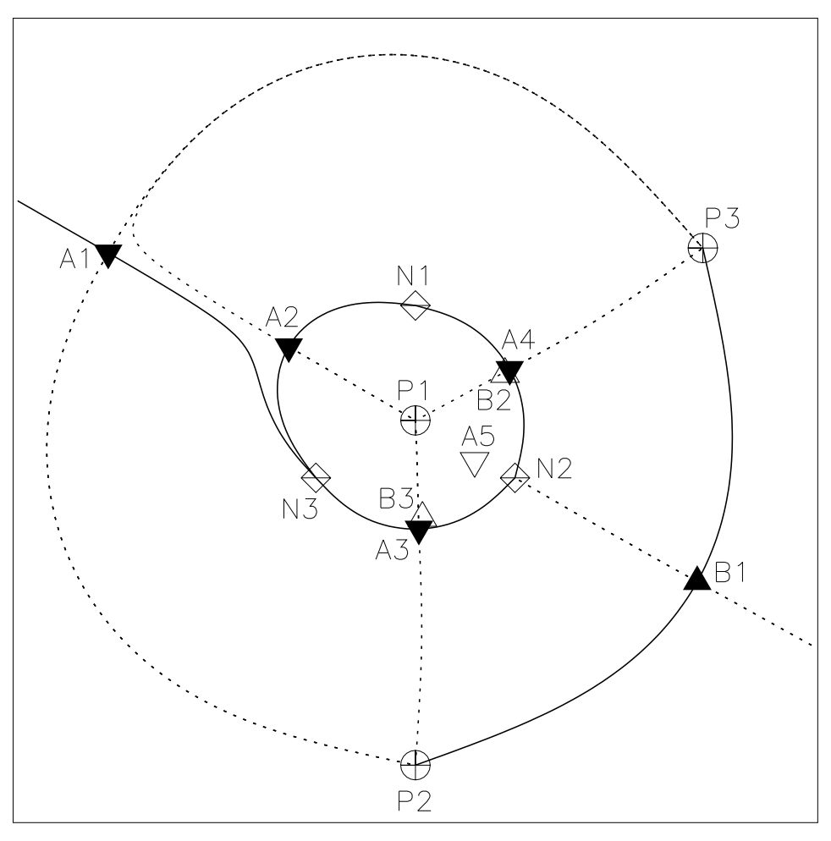
To model the delta sunspot, we assume that flux patches on the photosphere can be approximated by point magnetic charges (*sources*) on a flat plane (Longcope, 1996), with the corona as the half-space above.

In a potential field, this leads to a unique expression for the magnetic field $\mathbf{B}$. The positions of the important topological features of the field can then be calculated; amongst these are *null points* (where $\mathbf{B} = 0$), *spines* (fieldlines starting along the uniquely signed eigenvectors of the nulls), *separatrix surfaces* (sets of fieldlines starting in the surfaces defined by the other two eigenvectors of the nulls) and *separators* (fieldlines joining two null points).

Six sources are used in the model, with initial layout as shown in the diagram. Positive (negative) sources are labelled P (N); positive (negative) nulls are labelled B (A); spines are solid lines and separatrix traces are dotted lines. $P1$ represents the new flux emerging into the pre-existing sunspot. Magnetic breakout corresponds to flux from $P1$ connecting out to the balancing source at infinity, $N\infty$. In this initial state, the flux is blocked from doing so by the separatrix domes of $A1$, and $B2$ and $B3$.

Source	Position	Strength
P1	$(0, 0)$	ϵ
N1	$(0, 1)$	-1
N2	$(0.866, -0.5)$	-1
N3	$(-0.866, -0.5)$	-1
P2	$(0, -3)$	2.5
P3	$(2.5, 1.5)$	2.5

Initial source positions and strengths

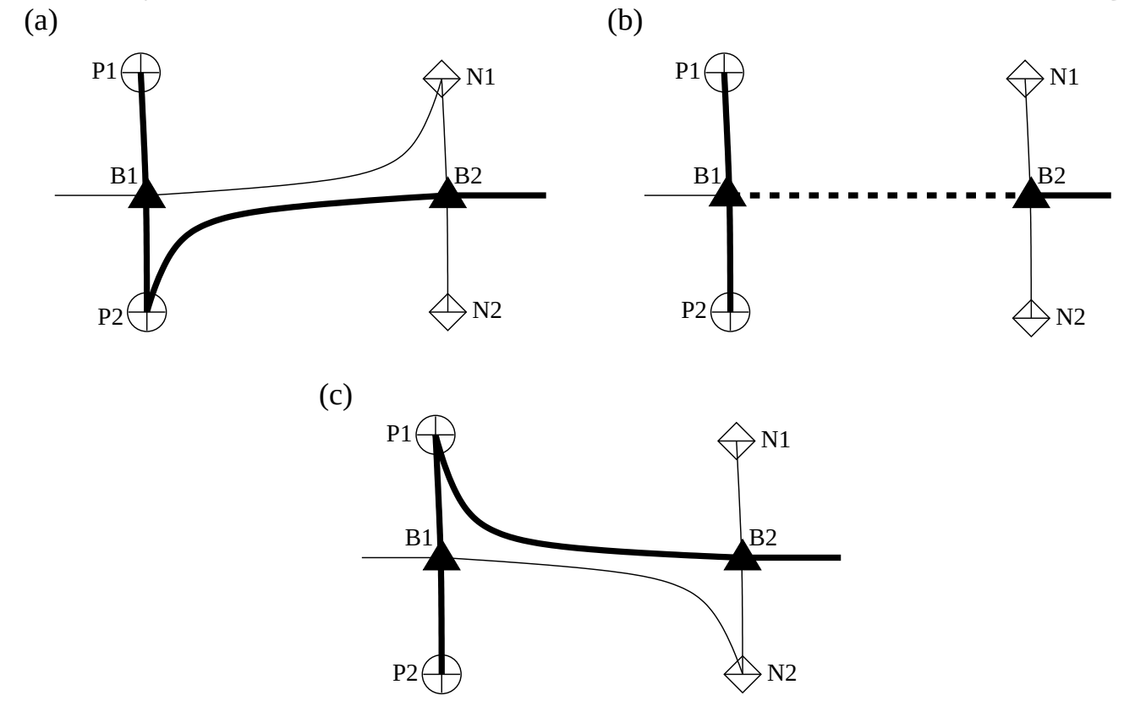


Magnetic footprint: plan view of initial topology

3) TOPOLOGICAL BIFURCATIONS

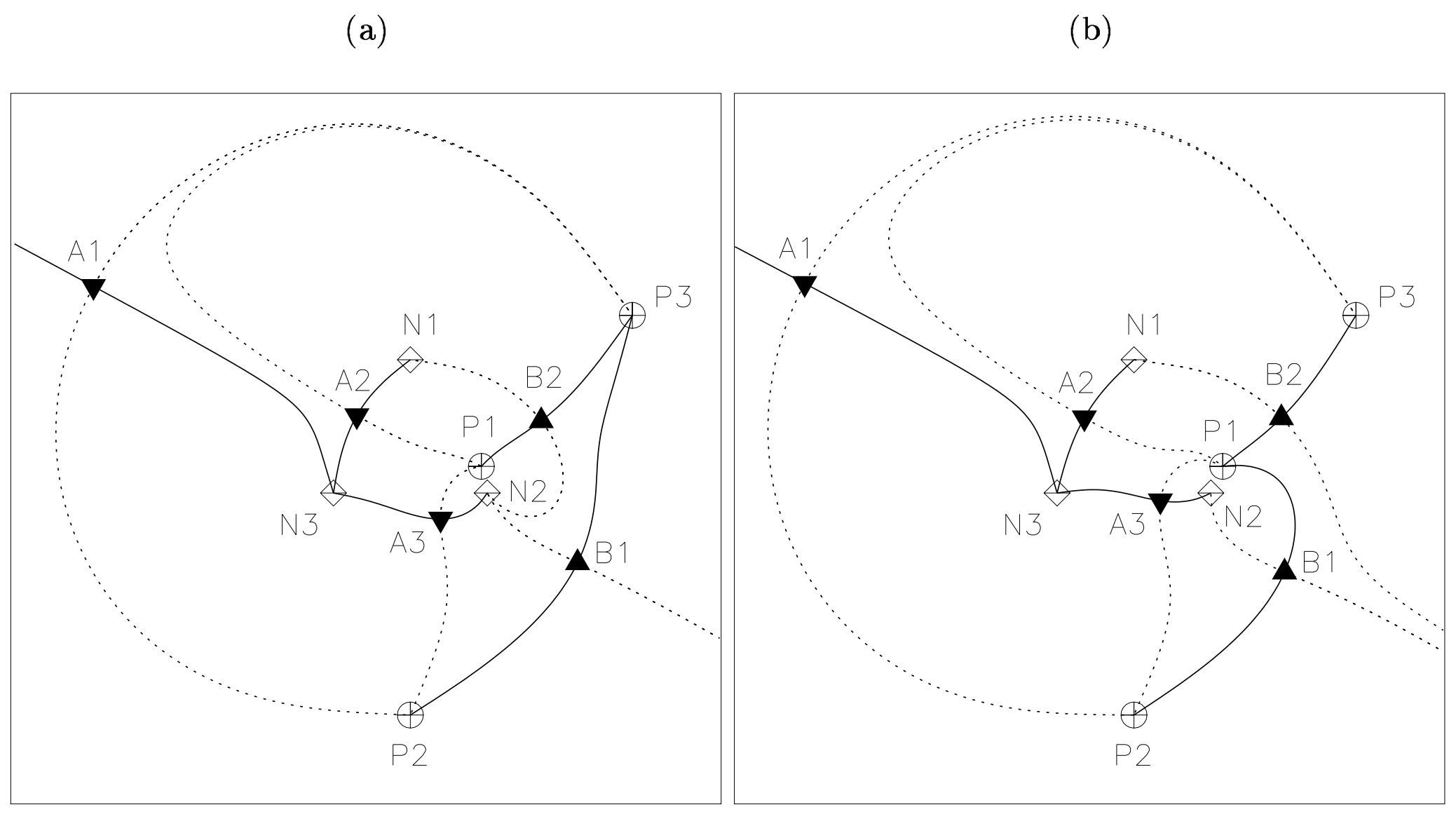
A *bifurcation* is a change in topological state. Two types of bifurcation are possible; *local* bifurcations in which the number of nulls changes, and *global* bifurcations in which there is a change in connectivity, but no nulls are created or destroyed.

Breakout can only be caused by global bifurcations, as only they can change the structure of the flux domains. A good example of this is the *global spine-fan bifurcation*, which is shown in the diagram below.



In a global spine-fan bifurcation, the spine of one null sweeps across the fan of a second, like-signed null. Here, the spine connecting null $B2$ to source $P2$ approaches null $B1$ (figure (a)). At the instant of bifurcation (figure (b)), the spine actually joins the fan thereby forming a separator connecting $B2$ to $B1$ in a structurally unstable topology. Immediately following this, the spine of $B2$ connects to $P1$ as shown in figure (c). The connectivity has changed.

4) EXAMPLE OF BREAKOUT: GLOBAL SPINE-FAN BIFURCATION



The global spine-fan bifurcation can cause breakout as shown in the example above. As $P1$ moves further right across the photosphere, the spine $B1$–$P3$ and the separatrix $B2$–$N2$ are pushed closer and closer together, until they coincide at about $x = 0.9$ in a global spine-fan bifurcation. After the bifurcation, the spine connects $B1$–$P1$ and the separatrix $B2$–$N\infty$. The bifurcation destroys the flux domain $P3$–$N2$, while at the same time creating a new flux domain $P1$–$N\infty$, the breakout domain.

5) TOPOLOGICAL TOOLS AND RULES

- *Domain* and *null* graphs: two sources are joined on the domain graph if they share flux, and two nulls are joined on the null graph if they are joined by a separator. Changes to the graphs help to determine which type of bifurcation has taken place.
- From Longcope and Klapper(2002), equations can be derived to describe the effect of bifurcations on the topology. For a local bifurcation,

  $$\Delta X = \Delta n_c + \tfrac{1}{2}\Delta n_\phi$$

  which relates the change in number of separators X to the change in number of coronal nulls n_c and photospheric nulls n_ϕ. For a global bifurcation,

  $$\Delta X = \Delta D_c + \tfrac{1}{2}\Delta D_\phi$$

  where D_c is the number of coronal domains and D_ϕ the number of photospheric domains.
- Separator rule for the global spine-fan bifurcation:
  If before a global spine-fan bifurcation, the null whose spine is involved (S) is connected to the set of nulls $\mathcal{S}'$ on the null graph, and the null whose fan is involved (T) is connected to the set $\mathcal{T}'$, then after the bifurcation, the set of nulls connected to T will be $(\mathcal{T}'\backslash\mathcal{S}') \cup (\mathcal{S}'\backslash\mathcal{T}')$.

6) PROVOKING BREAKOUT BEHAVIOUR

Three experiments were performed to provoke breakout behaviour:

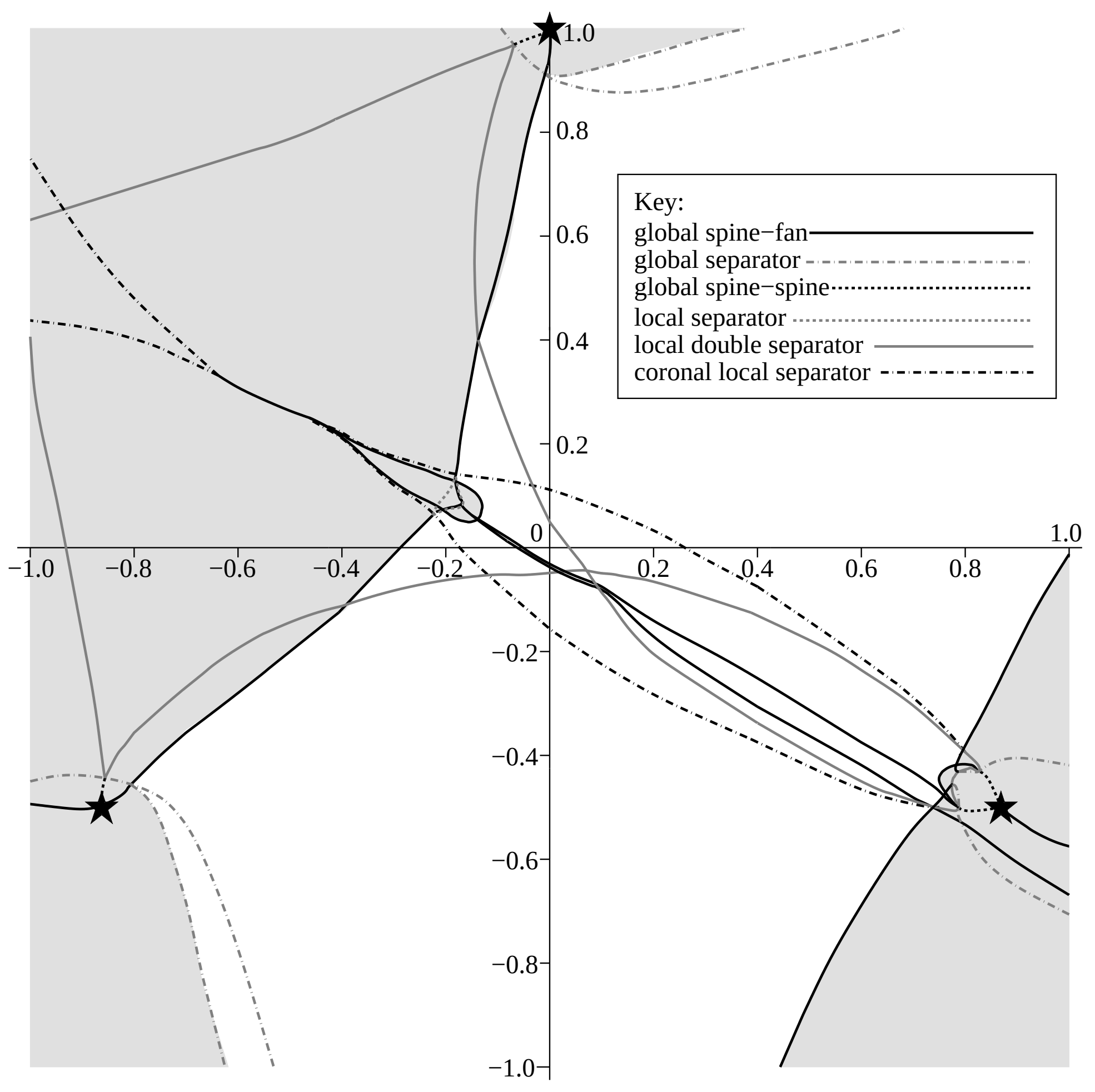
- altering the strength of the central source in a potential field, from just above 0 up to 2;
- altering the location of the central source in a potential field, within a 2×2 square centred on the origin; and
- altering the parameter α of a force-free field, while keeping $P1$ fixed near the origin with strength 1.5.

As the central source strength increases, a sequence of different local and global bifurcations takes place. When it reaches $\epsilon = 1.58$, there is a global spine-fan bifurcation which creates a flux domain between $P1$ and $N\infty$, giving magnetic breakout.

The bifurcation diagram created by altering the position of the central source is shown in panel 7. It shows that changing the source position leads to breakout in many directions. In fact, breakout would eventually be observed in all directions, as $P1$ moves far enough out to easily form a flux domain connecting itself to $N\infty$.

Both the *global spine-fan* and *global separator* bifurcations are found to lead to breakout topologies in this experiment. This result is important as it had previously been thought that only the global spine-fan bifurcation could cause breakout. The global separator bifurcation involves the creation or destruction of an intersection between two separatrix surfaces. Only global bifurcations can cause breakout as only they can create the new flux domain required to connect $P1$ to $N\infty$.

7) EFFECT OF SOURCE POSITION

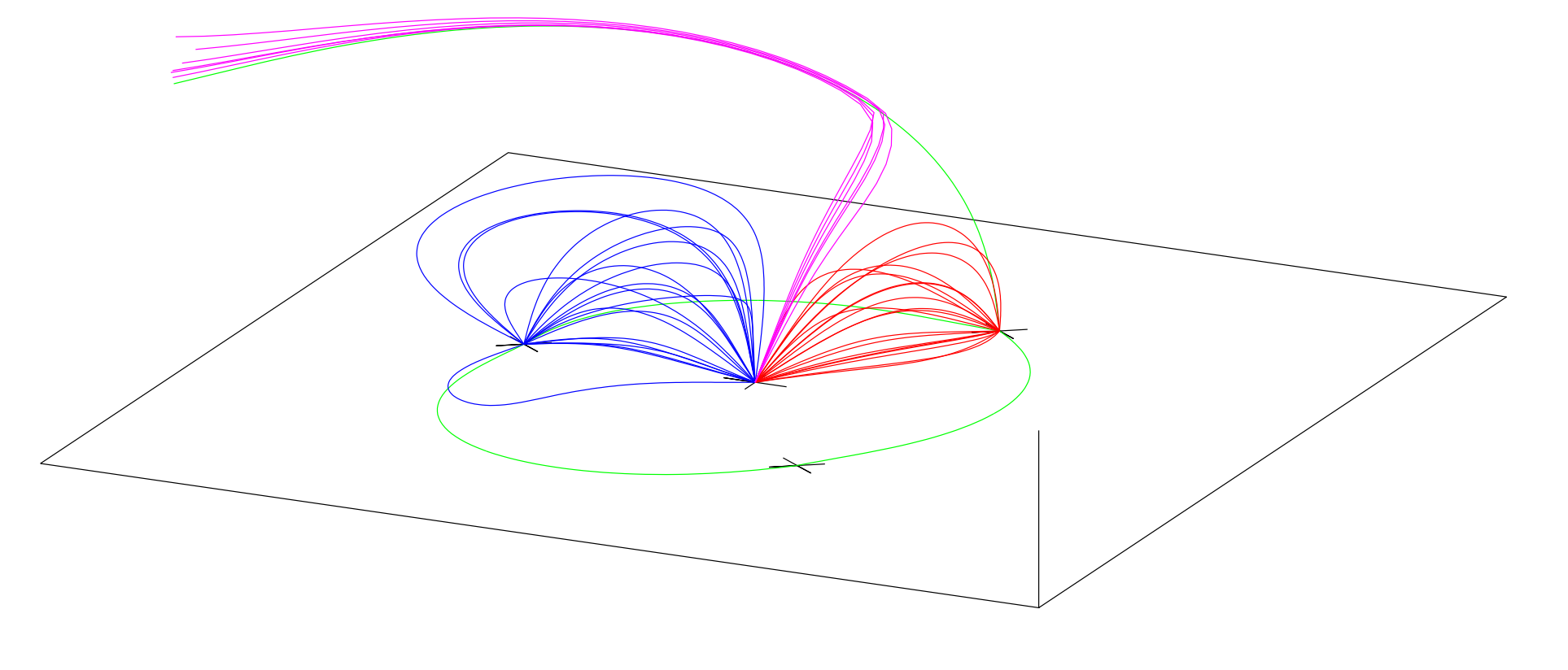


Bifurcation diagram as the central source is moved around a 2×2 box centred on the origin. Stars are sources; shaded areas indicate breakout topologies.

8) EFFECT OF FORCE-FREE PARAMETER α

A sequence of non-potential fields will exhibit similar types of topological changes to the potential fields considered so far. To demonstrate this we construct linear force-free fields for the same distribution of photospheric sources. The linear force-free field for a given α is computed by summing up the contributions of all sources, given by Green's functions (Chiu and Hilton, 1977). At distances beyond $\pi/2|\alpha|$, i.e. $r \simeq 7.5$, the radial field oscillates and the model becomes inaccurate. Hence our study is restricted to the field within this radius.

As the force-free parameter α increases from zero, some of the same bifurcations explored in the previous sections occur. At $\alpha = 0$ the topology is identical to that shown in Section 2. Three global spine-fan bifurcations occur as α is made increasingly negative, at $\alpha = -0.011, -0.028$ and -0.197. It is this third bifurcation which is responsible for breakout, by taking the spine of the negative coronal null to "infinity" and thus creating the breakout flux domain $P1$–$N\infty$. The manner in which the breakout proceeds is very similar to the previous potential field calculations, suggesting that a potential field gives a good qualitative picture of the topological behaviour of our sunspot.



3D view of topology after breakout: magenta fieldlines are in the breakout domain

9) CONCLUSIONS

The magnetic breakout model is far more complex than can be expressed with a potential field, accounting as it does for the energy storage necessary in the run-up to a solar flare, since potential fields are incapable of storing excess energy. However, even a potential field model of a delta sunspot can be made to display topological breakout behaviour in several distinct ways - by moving the flux sources or by altering the source strengths. A slightly more complicated, linear force-free field can also be made to "break out" by altering the parameter α.

It has been demonstrated that at least two different topological bifurcations can provide a mechanism for breakout, both of them global; the global spine-fan bifurcation and the global separator bifurcation. In fact, it seems that breakout behaviour is ubiquitous in our delta sunspot model; whichever parameter is varied, the system can eventually make its way towards a breakout configuration.

We hope that this work will pave the way for more realistic topological simulations of the magnetic breakout model.

Acknowledgements
We are grateful to the UK Particle Physics and Astronomy Research Council for funding.

References

Antiochos, S.K.: 1998, "The magnetic topology of solar eruptions", *Astrophys. J.*, **502**, L181-L184.

Antiochos, S.K., DeVore, C.R. and Klimchuk, J.A.: 1999, "A model for solar coronal mass ejections", *Astrophys. J.*, **510**, 485-493.

Chiu, Y.T. and Hilton, H.H.: 1977, "Exact Green's function method of solar force-free magnetic-field computations with constant alpha. I - Theory and basic test cases", *Astrophys. J.* **212**, 873-885.

Longcope, D.W.: 1996, "Topology and current ribbons: A model for current, reconnection and flaring in a complex, evolving corona", *Solar Phys.*, **169**, 91-121.

Longcope, D.W., and Klapper, I.: 2002, "A general theory of connectivity and current sheets in coronal magnetic fields anchored to discrete sources", *Astrophys. J.*, **579**, 468-481.

Tanaka, K.: 1991, "Studies on a very flare-active delta group - Peculiar delta SPOT evolution and inferred subsurface magnetic rope structure", *Solar Phys.* **136**, 133-149.

Contact: Rhona Maclean
Address: School of Mathematics and Statistics, University of St Andrews,
St Andrews, KY16 9SS, UK
Email: *rhonam@mcs.st-andrews.ac.uk*
WWW: `http://www-solar.mcs.st-andrews.ac.uk/`