Coronal Topological States in a Spherical Geometry

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Abstract

The topology of the coronal magnetic field is an important issue as it can explain the magnetic structures observed in the corona. We use a Green's function technique to extend the conventional methods of magnetic topology onto a spherical photosphere. Using four flux sources, bifurcation diagrams are generated showing the fundamental topologies which exist and how they are affected by changes in source size and position. A previously unknown state with two separators between a pair of null points is presented.

1) Introduction

The solar coronal magnetic field has been shown by observations to be highly complex. We wish to extend the proven techniques of magnetic charge topology, a powerful tool for analysis of magnetic fields, into a spherical geometry and thus apply them to the study of the global coronal field.

A necessary first step towards this goal is to understand the magnetic structures which occur in relatively simple fields, which can then act as building blocks in a full picture of the corona. Here, following the analysis of (1), we study the field produced in the corona by four point magnetic sources placed on the photospheric surface. We use a potential field for simplicity.

Two fundamental scenarios are possible: either two sources of each sign, or one of one sign and three of the other. We choose to study here the scenario of two bipoles, as it is more physically relevant. Several test cases were chosen for analysis in order to study the effects of changes in source strength and position on the resulting magnetic configuration. The details of these cases are shown in the table below.

	Source 1		Source 2		Source 3		Source 4	
Case	coords	ϵ	coords	ϵ	coords	ϵ	coords	ϵ
1	$(1,0,0)$	$+1$	$(1,\frac{2\pi}{3},0)$	$+0.6$	$(1,\frac{2\pi}{3},\frac{2\pi}{3})$	-1	$(1,\theta_1,\varphi_1)$	-0.6
2	$(1,0,0)$	$+1$	$(1,\frac{2\pi}{3},0)$	$+0.3$	$(1,\frac{2\pi}{3},\frac{2\pi}{3})$	-1	$(1,\theta_1,\varphi_1)$	-0.3
3	$(1,0,0)$	$+1$	$(1,\frac{2\pi}{3},0)$	$+0.9$	$(1,\frac{2\pi}{3},\frac{2\pi}{3})$	-1	$(1,\theta_1,\varphi_1)$	-0.9
4	$(1,0,0)$	$+1$	$(1,\frac{\pi}{4},0)$	$+0.6$	$(1,\frac{3\pi}{4},\frac{3\pi}{4})$	-1	$(1,\theta_1,\varphi_1)$	-0.6
5	$(1,0,0)$	$+1$	$(1,\frac{3\pi}{4},0)$	$+0.6$	$(1,\frac{\pi}{4},\frac{3\pi}{4})$	-1	$(1,\theta_1,\varphi_1)$	-0.6

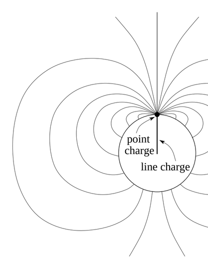
2) Green's Function Method

Given a set of point magnetic sources on the photosphere and the requirement that the photosphere should be a flux surface (i.e. no fieldlines should pass through it), a good method for constructing the potential magnetic field generated is not immediately obvious. As the method of spherical harmonics is inappropriate for use with point magnetic sources, we used a ***Green's function*** method instead. With the appropriate boundary conditions taken into consideration, the Green's function required for just one source is (4; 6)

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi}\left[\frac{2}{|\mathbf{r}-\mathbf{r}'|} - \ln\left(\frac{a^2 - \mathbf{r}\cdot\mathbf{r}' + a|\mathbf{r}-\mathbf{r}'|}{ra - \mathbf{r}\cdot\mathbf{r}'}\right)\right], \tag{1}$$

where $G(\mathbf{r},\mathbf{r}')$ is the Green's function, $\mathbf{r}$ is the position where we wish to calculate the field, and $\mathbf{r}'$ is the position vector of the source, which lies on the photosphere at radius a. Integration over the photosphere gives the scalar potential Φ, and then by the definition of a potential field, $\mathbf{B} = -\nabla\Phi$.

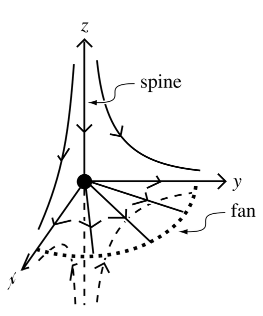
Physically, this Green's function represents a point source of charge $+2$ on the photosphere and a line source of integrated charge -1 extending from the centre of the sphere to touch its surface at the point charge. Hence from outside the sphere, the point source appears to have charge $+1$. The magnetic field strength is constant over the entire sphere, and the field has only a normal component on the surface. To cancel out this component, we place an oppositely signed balancing source elsewhere on the photosphere. Thus, the sources must always be balanced in order to keep the system physically realistic.



Sketch of magnetic fieldlines given by the Green's function for a charge at $\theta = 0$

3) Magnetic Charge Topology

The magnetic field produced by our set of point magnetic charges on the sphere will contain certain well-understood topological features, including *magnetic null points* where $\mathbf{B} = 0$. The field structure close to a first-order potential null is shown below.



The simplest structure of a generic potential magnetic null point

The expression for the field near the null can be linearised (5), so that the field is written as $\mathbf{B} = \mathbf{M}\cdot\mathbf{x}$, with

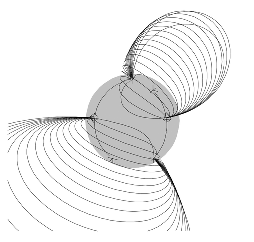
$$\mathbf{M} = \begin{pmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}q & 0 \\ \frac{1}{2}q & p & 0 \\ 0 & 0 & -(p+1) \end{pmatrix},$$

where p and q are parameters of the potential field. Calculating the eigenvalues and eigenvectors of $\mathbf{M}$, it can be shown that each null point has several special fieldlines associated with it. These are the isolated *spine* fieldlines, as well as the plane of *fan* fieldlines, which are always oppositely directed to the spine due to the requirement that $\nabla\cdot\mathbf{B} = 0$. As these fieldlines spread out into space, the fan planes form ***separatrices***, which are the boundaries between different ***flux domains*** – sets of fieldlines which are continuously deformable into one another and have the same source points.

The intersection of two separatrices creates a ***separator***; a fieldline joining two nulls of opposite sign. (A null point is known as ***positive*** if its fan fieldlines are directed away from the null, and ***negative*** otherwise). Separators form boundaries between four flux domains, and are the 3D analogue of the classic 2D X-point, as well as being prime locations for reconnection (3). Together, all of these features make up the ***skeleton*** of the ***topology*** under consideration.

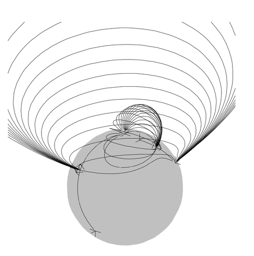
4) Topological States and Bifurcations

Five distinct topologies occur in the case of two sources of each sign. Corresponding examples of four of these topologies were observed by (1) in the planar photosphere case; one is new. An example of each is given below, along with a description of its distinctive features. In each plot, black lines are spine and separatrix fieldlines, stars are sources and polyhedra are nulls.



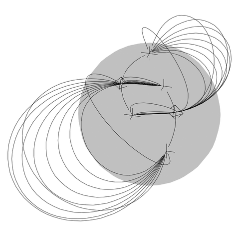
Detached state

The *detached* state involves two prone photospheric nulls, one of each sign. The spines of the positive (negative) null connect to the two positive (negative) sources, and all of the fieldlines in its separatrix surface go to one of the negative (positive) sources, forming a dome enclosing just one of its own spine sources. The two domes do not intersect one another, so there are three flux domains in this topology.



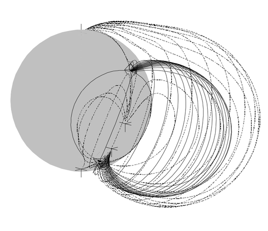
Nested state

The *nested* state is very similar to the detached state above; the difference between them lies in the arrangement of the separatrix surfaces. One remains unchanged (the smaller one in the diagram), enclosing one of its own spine sources. But the other has now changed which of its own spine sources it encloses, plus also covering the entire other separatrix dome. The two domes do not intersect, but are stacked up one on top of the other. Again there are three flux domains. In our later results, we refer to the nested state as a special case of the detached state.



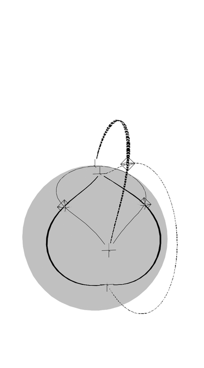
Intersecting state

In the *intersecting* state, the arrangement of the separatrices is different again. The separatrix of the positive (negative) null again forms a dome enclosing one of its own spine sources, but this time the dome touches both negative (positive) sources and the null between them. So the two domes intersect one another, creating a separator running between the two null points. This gives a total of four flux domains in the topology.



Dual intersecting state

The ***dual intersecting*** state is a new state, not previously found in the planar photosphere case of (1). Two spatially distinct separators join a pair of null points, a phenomenon never before observed in simple cases with only four sources. The two separators arise from the fact that the same two separatrix domes intersect one another twice. A source configuration close to being symmetrical is necessary to produce this state, but it is undoubtedly topologically stable. There are five flux domains in this topology. One is purely coronal, and engirdled by the loop formed by the two separators. Another two of the flux domains actually link the same two sources but are spatially separated from one another.



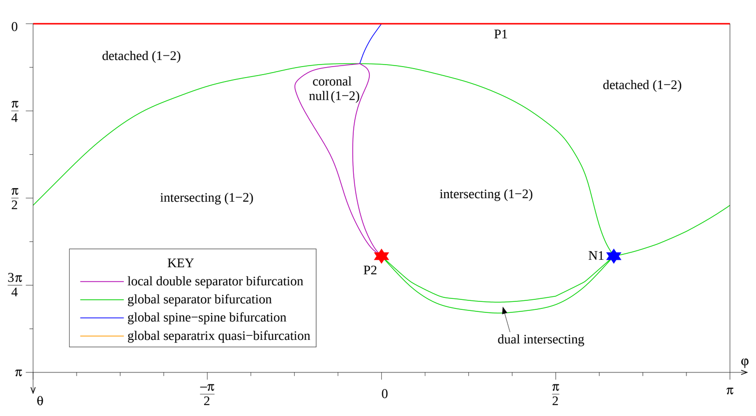
Coronal null state

The ***coronal null*** state has three null points; two of the same sign in the photosphere and one of opposite sign in the corona. Suppose that the coronal null is negative. Then the two photospheric nulls are positive, and their spines each connect to both positive sources, forming a loop in the photosphere around one of the negative sources. Similarly, each of their separatrices touches both negative sources, forming a dome around one of the positive sources. The separatrix of the coronal null also forms a dome, and the intersection of these two domes gives rise to separators between the coronal null and each of the photospheric ones. There are four flux domains.

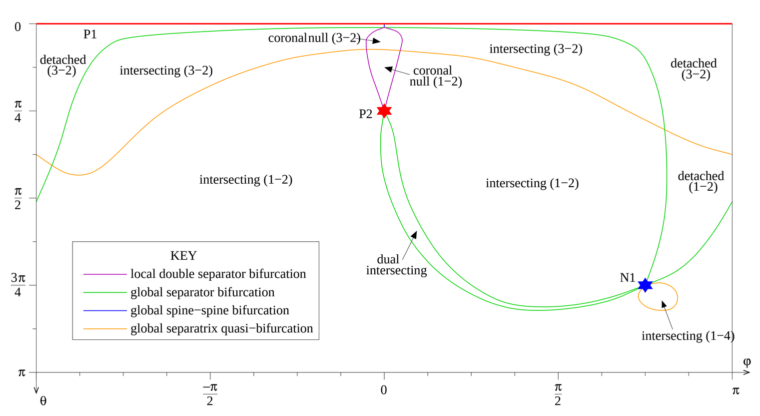
A ***bifurcation*** is a change from one topological state to another (2), caused here by variations in source strengths and positions. Three categories of bifurcation exist: ***local***, which change the number of null points, ***global***, which change the domain structure of the topology, and ***quasi-bifurcations***, which involve separatrices moving out to infinity and changing which flux domain overlies the others.

5) Bifurcation Diagrams

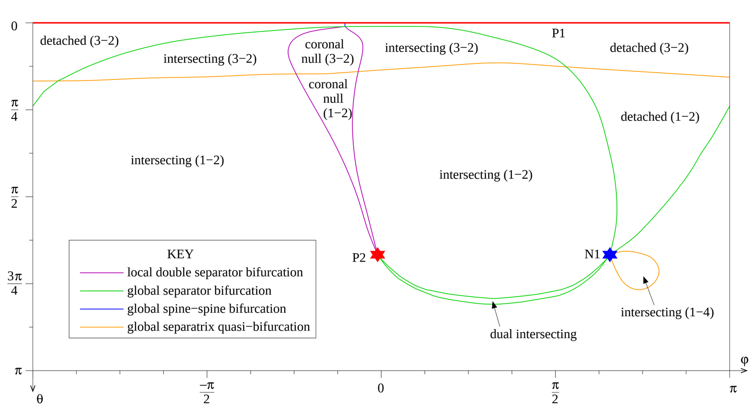
These are the bifurcation diagrams produced in the five cases which we studied. Each one shows the surface of the photosphere in spherical coordinates. Positive sources are coloured red and marked P, negative ones coloured blue and marked N. In each case P1 appears as a line rather than a point as it sits on the photosphere at $\theta = 0$. As the fourth source, N2, is moved over the surface, different topologies result. The regions where they occur are marked with their names. Numbers after a topology indicate which flux domain is the overlying one, the numbers being the labels of the two sources which create the flux domain.



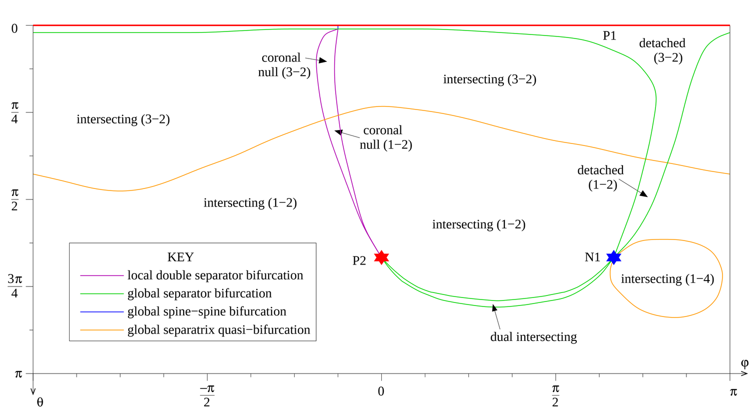
Case 2



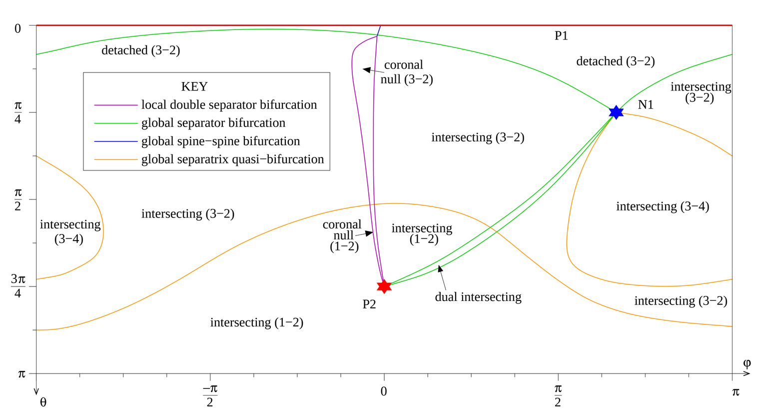
Case 4



Case 1



Case 3



Case 5

We take case 1 as our reference case, to which the other four can be compared. In case 2 the strengths of the weaker sources are decreased; this pushes the lines of the global separatrix quasi-bifurcations so close to sources that they become undetectable, while the detached states at the top become larger. The opposite is true in case 3 where the weaker sources are increased.

In cases 4 and 5, the strengths are the same as case 1 but the sources have changed position. We see essentially the same behaviour in each case, except that the dramatic changes in positioning of the quasi-bifurcations mean that the overlying flux domain is different, being mostly the domain (3-2) in case 5 and mostly (1-2) in case 4.

6) Conclusions

Four magnetic point sources on the photosphere can produce five distinct magnetic topologies in a potential field, one of which has not been previously found on a planar photosphere. This dual intersecting state involves a pair of separatrix surfaces intersecting each other twice, to produce a twinned pair of separators, which form a loop bounding a coronal domain. Two spatially separated flux domains join the same pair of sources. It had not been previously thought possible to find such an arrangement using so few point sources.

The arrangement of the topologies observed depends strongly on the strengths and positions of the sources which produce them. With one bipole much stronger than the other as in case 2, the quasi-bifurcations practically disappear and the detached state becomes more significant. Going the other way, with the bipoles almost equal in strength we see a global separatrix quasi-bifurcation dividing the globe into two almost equal parts, and the intersecting state dominating. Similarly, changing the position of the sources can change the area which each state covers on the sphere. As like sources move closer together, the stronger of the pair becomes more dominant, influencing the positioning of the quasi-bifurcation lines which are compressed in case 4 towards $\theta = 0$. Having unlike sources close together, on the other hand, leads to the dominance of the intersecting state and also allows flux domains from weaker sources to come into play much more as the stronger ones are cancelled out.

We hope that this work will lead to a better understanding of the global magnetic topology of the solar corona. In particular, we wish to apply these results to a topological study of the field structure during global field reversal.

References

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