Magnetic Skeletons: How to find them and why are they important

Andrew L. Haynes, Clare E. Parnell and Rhona C. Maclean

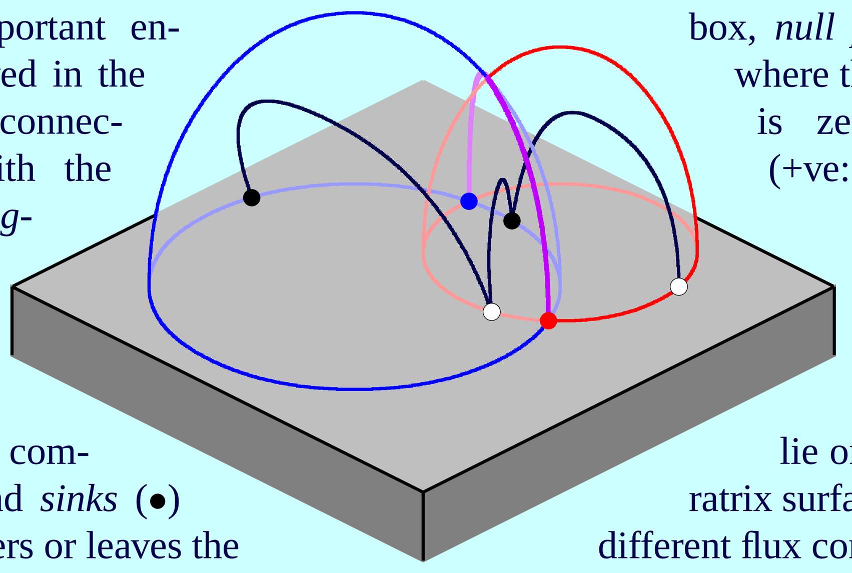
UNIVERSITY OF ST ANDREWS

ABSTRACT

The Sun is threaded by a complicated magnetic field crucial for understanding the transfer of energy between its interior and corona. One method of understanding it complex structure is to use the magnetic skeleton - a set of points (null points), surfaces (separatrix surfaces) and lines (separators) - which divides the magnetic field into many different volumes (flux domains), each containing flux with the same connections. The magnetic skeleton is important because it identifies many of the key locations in the magnetic field that are likely sites for magnetic reconnection. Although the definition of the magnetic skeleton is relatively simple, in practice it is difficult to find the full magnetic skeleton. Here, we present one method to find the magnetic skeleton and apply this method to a number of examples. One such example is of a numerical resistive MHD model of flux emergence. This will show for the first time that the magnetic skeleton in flux emergence is likely to be incredibly complicated and related to some of the most important locations of magnetic reconnection and energy release in these models.

1. Introduction

Some of the most important energy release sites involved in the process of magnetic reconnection are associated with the components of the *magnetic skeleton* (right), which is itself defined by the magnetic topology of the field. The magnetic skeleton comprises of *sources* (○) and *sinks* (●) where magnetic flux enters or leaves the box, *null points* (+ve: red ●, −ve: blue ●) where the magnetic field strength is zero, *separatrix surfaces* (+ve: red —, -ve: blue —) which extend from null points and divide flux systems connected to different sources (sinks) and *separators* (purple —) which lie on the intersection of separatrix surfaces and hence divide four different flux connectivities.



2. Finding Null Points

Trilinear Assumption: We assume that the field within any grid cell is trilinear, thus can be written in the form

$$\mathbf{B} = \mathbf{a} + \mathbf{b}x + \mathbf{c}y + \mathbf{d}xy + \mathbf{e}z + \mathbf{f}xz + \mathbf{g}yz + \mathbf{h}xyz$$

The coefficients $\mathbf{a}$ to $\mathbf{h}$ may be found from the values of the magnetic field at the vertices.

Method

Reduction: Remove all cells where at least one component of the field has the same sign at all vertices, as this cell cannot contain a null point.

Analysis: Consider the curves of $B_x = 0$, $B_y = 0$ and $B_z = 0$ on the bilinear cell boundaries. A null point exists if, and only if, each pair of curves must intersect at least twice and the other component changes sign between these two points.

Positioning: The exact location of the null point is found using a Newton-Raphson method:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - [\nabla \mathbf{B}(\mathbf{x}_n)]^{-1} \mathbf{B}(\mathbf{x}_n)$$

Multiple choices of $\mathbf{x}_0$ may be required to find a solution within the grid cell.

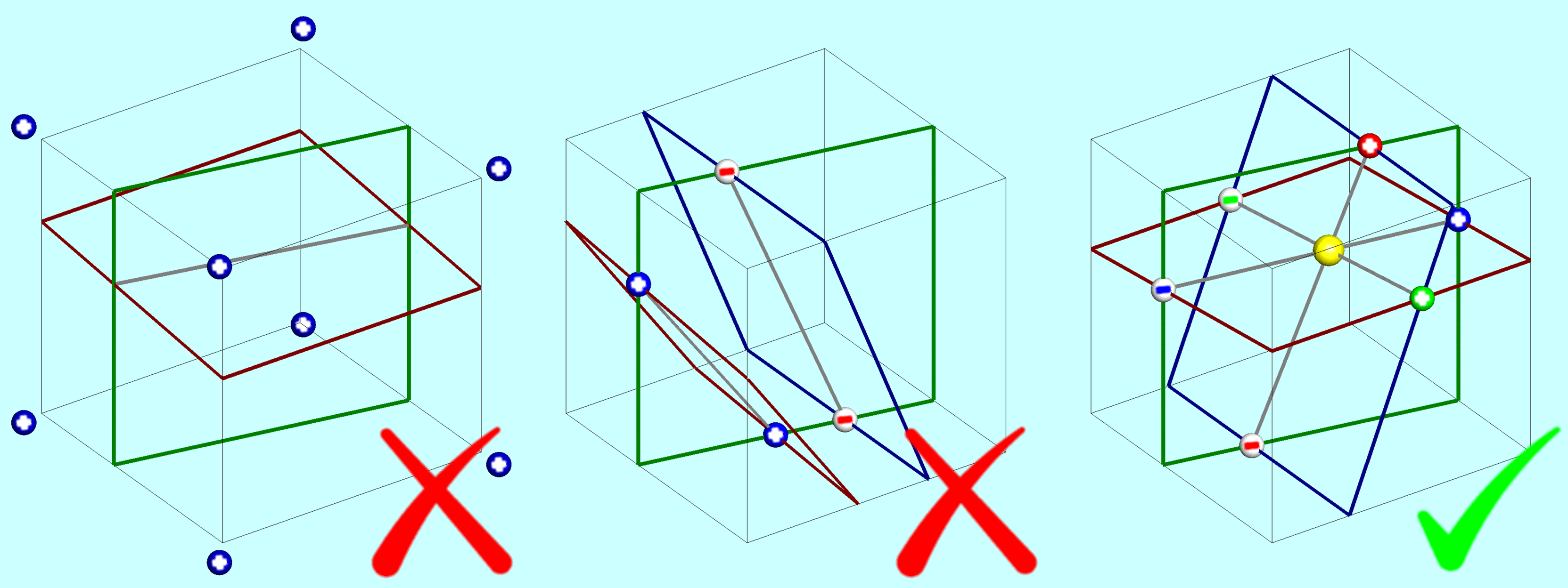
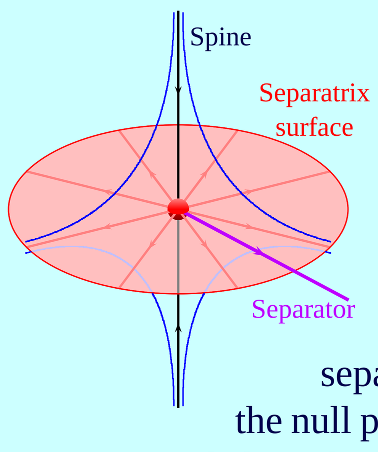


Figure 1. Three example grid cells with the surfaces of $B_x = 0$ (red), $B_y = 0$ (green) and $B_z = 0$ (blue) shown by their intersections with the cell boundary. The spheres denote the sign of a component (coloured as surfaces) at the vertices (left) or the ends of the grey intersection lines (centre and right). The null is a yellow sphere. The left example fails the reduction step (as the vertices all show positive B_z) and the centre fails the analysis phase (as the third component, either B_x or B_z, does not change sign on the grey intersection curves). The right is an example where the null point is found.

3. Finding Separatrix Surfaces and Separators



Method

At each null point, the starting points of the separatrix surfaces are found from the eigenvalues and eigenvectors of the magnetic field. All generic null points have two eigenvalues of the same sign, and their associated eigenvectors define the separatrix surface on a ring of small radius near the null point (see left; Parnell et al., 1996). The final eigenvector defines the spines.

Separatrix Surfaces: The separatrix surface algorithm then traces out the separatrix surface from the outermost ring onto a new ring, interpolating new points to fill in the gaps. Should a null point exist between two of the points on the ring, then the ring is broken at the null point and the ring points on either side of the null point set to locations on the spines from the null point. The algorithm stops when the outer ring has completely left the domain.

Separators: Whenever one of the rings from a null point is broken at a different null point, a line is traced back through the rings to the original null point. The full set of lines defines all the separators extending from the null point and lying wholly within the domain.

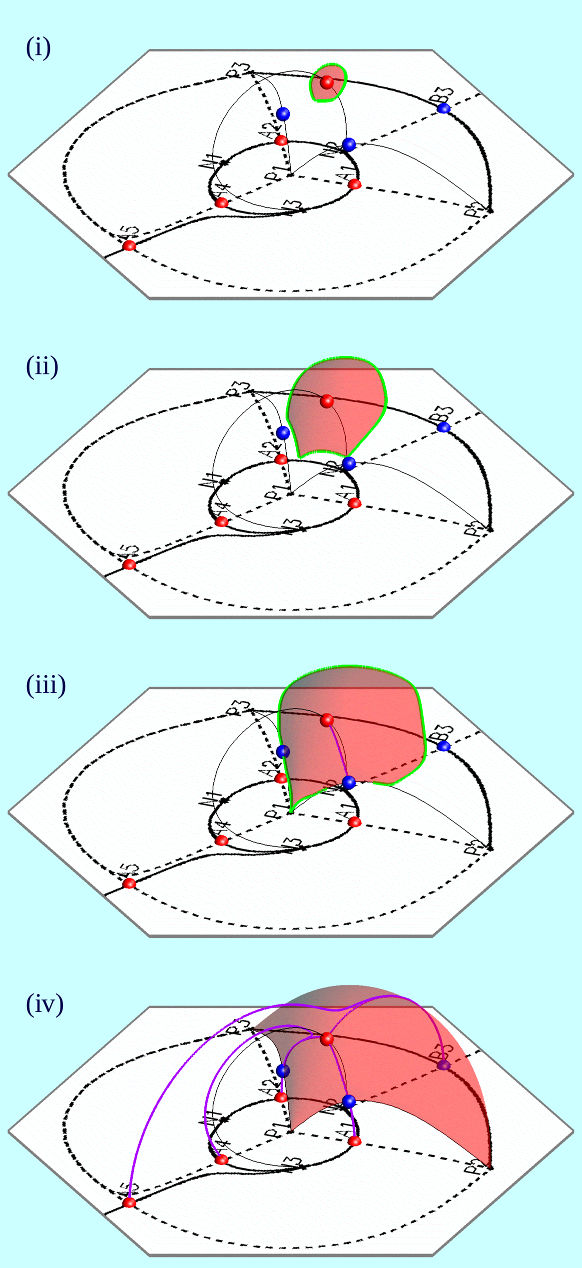


Figure 2. (right) Four stages of the separatrix surface and separator finding algorithm. In this example, we have a potential magnetic field generated from three positive and three negative point sources (+ and ×). These generate five positive and three negative null points (red and blue spheres). The images show (i) the initial ring (green) found from the eigenvalues and eigenvectors of the coronal positive null point, (ii) the expanding separatrix surface (red with green outer ring) before breaking at any null point, (iii) the expanding separatrix surface after breaking at the two negative coronal null points and the separators found from these breaks and (iv) the complete separatrix surface and all separators in the model.

4. Example I: Magnetic Flybys

Magnetic flybys (Galsgaard et al., 2000) occur when two magnetic field fragments on the photosphere beneath an overlying magnetic field pass one-another without loss or gain of magnetic flux. Energy release occurs when the flux lobes from each fragment collide and then reconnect to allow the lobes to pass through each other.

Evolution: By finding the magnetic skeleton (Haynes et al., 2007), we were able to find a series of seven phases of the experiment, each with a different magnetic skeleton. Initially, the sources are unconnected and only two null points on the photosphere (and their separatrix surface) are present. Then the model heads through phases with 2, 1, 5, 3 and 1 separators before the sources disconnect, during which magnetic flux closes (connects the sources) or opens.

Reconnection: Many of the magnetic separators thread regions of high current which have been shown to be associated with separator reconnection. The presence of multiple separators between a pair of null points leads to recursive reconnection (Parnell et al., 2008), a process that allows magnetic flux to repeatedly close and reopen multiple times, hence allowing more reconnection that previously expected.

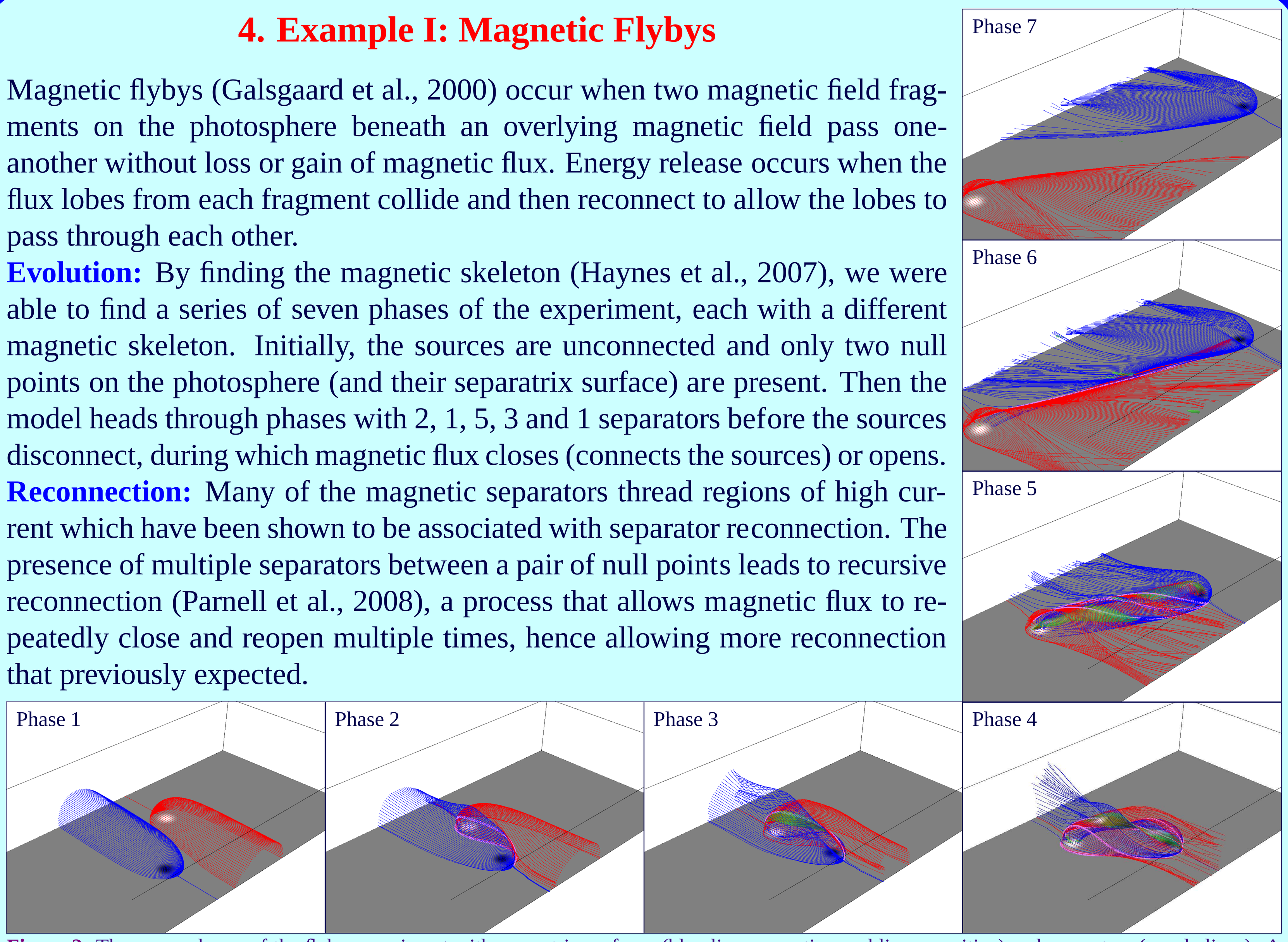


Figure 3. The seven phases of the flyby experiment with separatrix surfaces (blue lines: negative; red lines: positive) and separators (purple lines). A isosurface of current (green) is also shown.

5. Example II: Flux Emergence

The second example analyses a model of flux emergence (Galsgaard et al., 2007) where a twisted magnetic flux tube rises from beneath the photosphere and up into the atmosphere to reconnect with the overlying coronal magnetic field.

Null Points: Maclean et al. (2009) considered and explained the creation, evolution and destruction of all null points throughout the experiment. They found that the null points occurred in two clusters on either side of the flux tube. Up to 26 null points per frame were found but the null points did not coincide with the current sheets (and reconnection sites).

Separators: We expanded this research to find the separators connecting all the null points. A high number of separators were found, with an initially complex skeleton with many separators linking the two null clusters. Later, there are three separators between the clusters (one high and two pulled under the photosphere). Many of the separators thread the current sheets, suggesting that separator reconnection is important for the global restructuring of the magnetic field in this experiment.

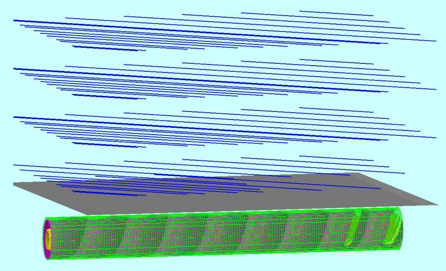


Figure 4. Initial setup with fieldlines (blue: overlying; yellow/purple: flux tube), isosurface of current (green) and photosphere (grey).

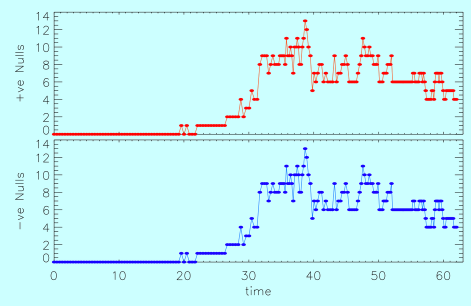


Figure 5. Number of positive and negative null points throughout the experiment (Maclean et al., 2009)

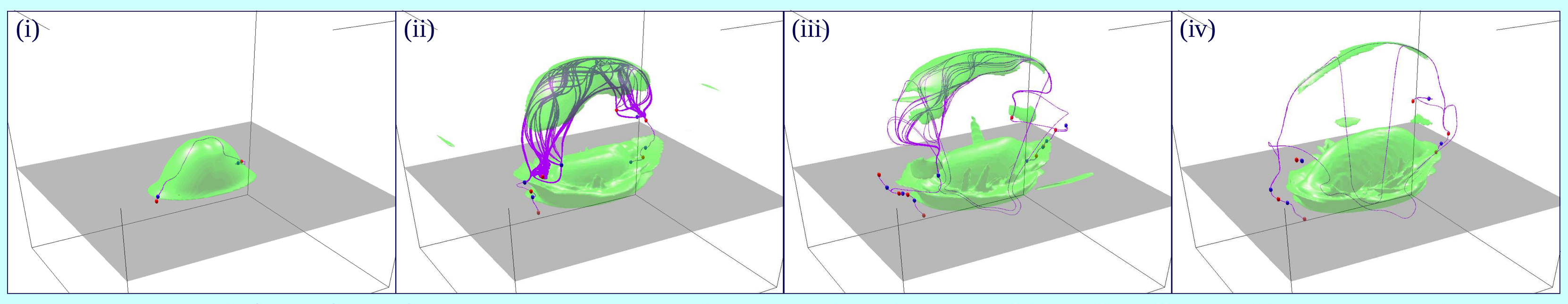


Figure 6. Four sample frames from a flux emergence experiment taken at $t = 27.1$, 34.6, 38.0 and 42.9. The null points (blue: negative; red: positive), separators (purple) and an isosurface of current (green) are plotted.

6. Conclusions

1. We have devised new methods for finding the magnetic skeleton, suitable for complex magnetic fields generated by numerical-MHD experiments.
2. We have demonstrated this new method on two numerical experiments.
   - Each of these gives a rich and complex magnetic skeleton with many separators connecting the same pair of null points.
   - Some of the separators are clearly associated with current sheets, suggesting that separator reconnection at these separators is important for the restructuring of the global magnetic field.

References

Galsgaard, K., Archontis, V., Moreno-Insertis, F., and Hood, A. W. (2007). The Effect of the Relative Orientation between the Coronal Field and New Emerging Flux. I. Global Properties. *Astrophys. J.*, 666:516–531.

Galsgaard, K., Parnell, C. E., and Blaizot, J. (2000). Elementary heating events - Magnetic interactions between two flux sources. *Astron. Astrophys.*, 362:395–405.

Haynes, A. L. and Parnell, C. E. (2007). A trilinear method for finding null points in a three-dimensional vector space. *Physics of Plasmas*, 14(8):082107.

Haynes, A. L., Parnell, C. E., Galsgaard, K., and Priest, E. R. (2007). Magnetohydrodynamic evolution of magnetic skeletons. *Proc. Royal Soc. A*, 463:1097–1115.

Maclean, R. C., Parnell, C. E., and Galsgaard, K. (2009). Is null-point reconnection important for solar flux emergence? *Solar Phys. (in press)*.

Parnell, C. E., Haynes, A. L., and Galsgaard, K. (2008). Recursive Reconnection and Magnetic Skeletons. *Astrophys. J.*, 675:1656–1665.

Parnell, C. E., Smith, J. M., Neukirch, T., and Priest, E. R. (1996). The structure of three-dimensional magnetic neutral points. *Physics of Plasmas*, 3:759–770.

Contact: Andrew L. Haynes
Address: Mathematics Institute, University of St Andrews, St Andrews, KY16 9SS, UK
e-mail: *andrew@mcs.st-andrews.ac.uk*
www: http://www-solar.mcs.st-andrews.ac.uk/~andrew/