MHD Waves in Earth’s Magnetosphere: My understanding thus far

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Overview

- The magnetosphere, observations:
  1. The Dungey Cycle
  2. MHD/ULF Waves
  3. Resonance
  4. Fast/Alfvén Modes

- Theory:
  1. MHD, cold plasma equations
  2. Wave equations
  3. Numerics
  4. My work (if time!)
Wave Modes

Resonant Magnetic Field Line

Cluster
Kelvin-Helmholtz Surface Wave

Magnetopause

Solar Wind
Ground magnetometer data

Clausen et al., [2008]
Satellite data

Hartinger et al., [2012]
- Dungey cycle $\Rightarrow$ magnetospheric convection, aurora etc

- Dayside compressional (fast) modes

- Field line resonance (FLR) ie the toroidal Alfvén wave
The start of the theory - box geometry
Ideal low beta plasma (low pressure).

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B), \]

\[ \rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla) u = j \times B - \nabla p + \rho g, \]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0. \]
Equilibrium is given by $\mathbf{B} = B_0 \mathbf{\hat{z}}$, $\mathbf{u}_0 = 0$, $\rho = \rho_0(x)$.

\[
\begin{align*}
\frac{\partial b_x}{\partial t} &= B_0 \frac{\partial u_x}{\partial z}, \\
\frac{\partial b_y}{\partial t} &= B_0 \frac{\partial u_y}{\partial z}, \\
\frac{\partial b_z}{\partial t} &= -B_0 \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right), \\
\rho_0 \frac{\partial u_x}{\partial t} &= \left( \frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x} \right) \frac{B_0}{\mu}, \\
\rho_0 \frac{\partial u_y}{\partial t} &= \left( \frac{\partial b_y}{\partial z} - \frac{\partial b_z}{\partial y} \right) \frac{B_0}{\mu}, \\
\frac{\partial u_z}{\partial t} &= 0.
\end{align*}
\]
Assume a standing mode in $\hat{z}$.

\[
\begin{align*}
\frac{\partial b_x}{\partial t} &= -k_z u_x, \\
\frac{\partial b_y}{\partial t} &= -k_z u_y, \\
\frac{\partial b_z}{\partial t} &= -\left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right), \\
\frac{\partial u_x}{\partial t} &= \frac{1}{\rho_0} \left( k_z b_x - \frac{\partial b_z}{\partial x} \right), \\
\frac{\partial u_y}{\partial t} &= \frac{1}{\rho_0} \left( k_z b_y - \frac{\partial b_z}{\partial y} \right).
\end{align*}
\]

which we solve with a Leapfrog-trapezoidal finite difference scheme of Zalesak, [1979].
Equations (for understanding)

\[ \frac{1}{V_A^2} \frac{\partial^2 \xi_x}{\partial t^2} - \frac{\partial^2 \xi_x}{\partial x^2} = -\frac{1}{B} \frac{\partial b_z}{\partial x}, \tag{1} \]

\[ \frac{1}{V_A^2} \frac{\partial^2 \xi_y}{\partial t^2} - \frac{\partial^2 \xi_y}{\partial z^2} = -\frac{1}{B} \frac{\partial b_z}{\partial y}, \tag{2} \]

\[ b_z = -B \left( \frac{\partial \xi_x}{\partial x} + \frac{\partial \xi_y}{\partial y} \right). \tag{3} \]

- Alfvén resonance growth in $\xi_y$.
- Driven through magnetic pressure gradient of fast mode in y direction.
- LHS of (2) is a simple harmonic oscillator and RHS is the driver.
Fourier analysis (seek normal modes), assuming perturbations of the form $e^{i(\omega t \pm ky \pm kz)}$

$$\frac{d^2 b_z}{dx^2} - \frac{\omega^2 V_A^{-2} dV_A}{\omega^2/V_A^2 - k_z^2} \cdot \frac{db_z}{dx} + \left( \frac{\omega^2}{V_A^2} - k_y^2 - k_z^2 \right) b_z = 0$$

Singularity when $\omega^2 = V_A^2 k_z^2$, at a specific location in $x$.

Represents location where energy from longitudinal waves is converted into transverse waves.
Efficiency depends on $k_y$.

If $k_y$ too small, the gradients aren’t large enough to drive transverse displacements.

If $k_y$ too large, the amplitudes in the resonant tail of the fast mode aren’t large enough.

Most effective for medium $k_y$.

One of Andy’s main works: showing that near $k_y = 0$ are the most effective drivers of Alfvén resonances.
Consider modelling specific observations of fast/Alfvén waves.

Use a numerical model to solve cold plasma equations.

Developed a new boundary condition for the driven boundary.

Try to get more from the data, things we’ve shown are locations of source regions, length of driving periods and also negating claims of FLR observations.
Finite Difference Method

Can express equations in the form

\[
\frac{\partial \mathbf{U}}{\partial t} = \mathbf{F}
\]

where

\[
\mathbf{U} = \begin{pmatrix}
u_x \\ u_y \\ b_x \\ b_y \\ b_z
\end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix}
(k_z b_x - b_z, x) / \rho \\
(k_z b_y - b_z, y) / \rho \\
-k_z u_x \\
-k_z u_y \\
-(u_x, x + u_y, y)
\end{pmatrix}
\]
Assuming we know $U$ at times $t$ and $t - \Delta t$, then the scheme is

\[
U^\dagger = U^{t-\Delta t} + 2\Delta t F^t \\
F^* = \frac{1}{2} (F^t + F^\dagger), \\
U^{t+\Delta t} = U^t + \Delta t F^*.
\]

- Use centered finite differences to calculate the spatial derivatives.
- Scheme is second order accurate in time and space.
- May require another algorithm to get started if 2 timesteps are not known.
Discussed global magnetospheric processes.

Showed observational wave evidence and notional ideas of waves.

Modelling ULF waves from an MHD perspective.

Concepts of resonance and fast mode driving in terms of equations.

Touched on the numerics.

All of the above is the background to what I study!