Field Interpolation*

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1 Motivation

We seek to classify null points as positive or negative. From the theory discussed elsewhere, the first task is evidently to determine the Jacobian of the field at the position of the null point in question. For the first element in the matrix, we can obviously write

$$\frac{\partial B_x}{\partial x} \approx \frac{B_x(x_0 + \delta x, y_0, z_0) - B_x(x_0 - \delta x, y_0, z_0)}{2\delta x}$$

for some small perturbation δx. But clearly the field is only known at discrete points in the volume, and to set δx to the width of an entire pixel is unacceptable. In order to determine any of the Jacobian derivatives, we must use values of the field in the immediate vicinity of the relevant point. Since those values are unknown, we must interpolate the field.

In each case in this discussion, we shall consider just the x-component of the field.

2 1d interpolation

First, we consider the 1d case. The simplest form of interpolation between two known values on a line, $B_x(0)$ and $B_x(1)$, generates the equation

$$B_x(x) = B_x(0) + (B_x(1) - B_x(0))x$$

which we can rewrite

$$B_x = (1 - x)b_0 + xb_1$$

3 2d interpolation

This is easily expanded to 2d. Points along the bottom horizontal line are given by $b_0(x) = (1 - x)b_{00} + xb_{10}$, and along the top horizontal line by $b_1(x) = (1 - x)b_{01} + xb_{11}$, and the point at (x, y) is given by interpolating along the vertical line of constant x between $b_0(x)$ and $b_1(x)$, ie.

$$B_x(x, y) = (1 - y)b_0(x) + yb_1(x) = (1 - x)(1 - y)b_{00} + x(1 - y)b_{10} + (1 - x)yb_{01} + xyb_{11}$$

*Using the linear interpolation theory expounded by [Haynes Parnell 2000].

which we can rewrite

$$B_x(x, y) = a + bx + cy + dxy$$

where $a = f_{00}, b = f_{10} - f_{00}, c = f_{01} - f_{00}, d = f_{11} - f_{10} - f_{01} + f_{00}$.

4 3d interpolation

Similarly, a trilinear equation can be obtained for the 3d case,

$$B_x(x, y, z) = a + bx + cy + dxy + ez + fxz + gyz + hxyz$$

where,

$$\begin{aligned} a &= b_{000} & b &= b_{100} - b_{000} \\ c &= b_{010} - b_{000} & d &= b_{110} - b_{100} - b_{010} + b_{000} \\ e &= b_{001} - b_{000} & f &= b_{101} - b_{100} - b_{001} + b_{000} \\ g &= b_{011} - b_{010} - b_{001} + b_{000} & h &= b_{111} - b_{110} - b_{101} - b_{011} - b_{100} + b_{010} + b_{001} - b_{000} \end{aligned}$$

5 Calculating the Jacobian

Our approach, then, for determining the field strength at some arbitrary point P in the field is to form what we shall call a 'capture cube' about P, where each corner is a point where the field strength is known directly from the available data. We interpolate the field using this cube to deduce its approximate value at P. Since we are attempting to calculate the Jacobian at a null point it will be necessary to determine the field strength for positive and negative perturbations about the null in x, y and z. This may necessitate the use of more than one capture cube, depending on whether or not the point in question lies on a face or in one of the corners of its associated cube.